Quantum Noise in Resistive Mixers

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ABSTRACT

Release E : Derivation of the quantum noise of a mixer using second order quantization methods

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1. QUANTUM THEORY OF DIRECT DETECTION

In order to establish some necessary relations, let us consider first a quantum physical approach to direct detection. There the receiver (i.e. a diode) generates a certain amount of photoelectrons per incident detected photon. The quantum efficiency η is assumed unity in the following sections. The detection of the incident radiation is equivalent to the counting operator of the incident photons. The charge operator $\hat{Q}$ governing the photon absorption and ideal photoelectron generation in a diode is given by the scalar product of a field operator $\hat{A}$ with its hermitian applied to any initial and final quantum state α . This scalar product yields the number of photoelectrons when multiplying the scalar product with the carrier charge q.

$$\hat{Q} = q\hat{A}^{\dagger}\hat{A} \tag{1}$$

Then the net charge $\langle\hat{Q}\rangle$ absorbed in the diode during a coherent state α becomes:

$$\langle\hat{Q}\rangle := \langle\alpha|\hat{Q}|\alpha\rangle = q\langle\alpha|\hat{A}^{\dagger}\hat{A}|\alpha\rangle = q|\alpha|^2 = q\langle n\rangle \tag{2}$$

Applying the commutator on the numbering operator, we can calculate the second order moment after sorting the operators in creators and destructors as follows:

$$\langle\hat{Q}^2\rangle := \langle\alpha|\hat{Q}^2|\alpha\rangle = q^2\langle\alpha|\hat{A}^{\dagger}\hat{A}\hat{A}^{\dagger}\hat{A}|\alpha\rangle = q^2|\alpha|^4 + q^2|\alpha|^2 = q^2\langle n\rangle^2 + q^2\langle n\rangle \tag{3}$$

$$\langle\hat{Q}^2\rangle - \langle\hat{Q}\rangle^2 = q^2\langle n\rangle \tag{4}$$

It is important to note that the squared variance of the charge absorbed in the diode is equal to the number density indicating that the photon and the subsequent electron flow is completely uncorrelated. As a consequence there is a one-to-one correspondence between the photon flux and the flux of the generated photoelectrons. As we will see later, this is the major difference between a diode and a bolometric receiver where the photons "integrate up" and create a hot spot. This causes the variance to be only half a RF quantum in the limit of an infinitely slow bolometer whereas the mean value is obtained in the same way as for the diode.

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2. COHERENT QUANTUM LIMITED HETERODYNE DETECTION

Superimposing a Local Oscillator and a Radio Frequency Signal one arrives at a field in front of the mixer as:

$$\hat{B} = \hat{A}_{LO} + \hat{A}_{RF}; \hat{A}_{LO(RF)} = \alpha_{LO(RF)} e^{i \cdot \omega_{LO(RF)} \cdot t} \tag{5}$$

For an non-balanced heterodyne detection, the charge generated in a detector is similar to 1 for a small RF signal compared to the LO:

$$\hat{Q} = q \cdot \hat{B}^{\dagger} \hat{B} = q \cdot (\hat{A}_{LO} + \hat{A}_{RF})^{\dagger} \cdot (\hat{A}_{LO} + \hat{A}_{RF}) \approx q \cdot (\hat{A}_{LO}^{\dagger} \hat{A}_{LO} + \hat{A}_{RF}^{\dagger} \hat{A}_{LO} + \hat{A}_{LO}^{\dagger} \hat{A}_{RF}) \tag{6}$$

The mean value of the photons observed by the detector is thus:

$$\langle Q \rangle = 2|\alpha_{LO}\alpha_{RF}| \cos((\omega_{RF} - \omega_{LO})t) + |\alpha_{LO}|^2 + |\alpha_{RF}|^2 \tag{7}$$

On our way to calculate the quantum noise of the detector, we have to evaluate the second moment of the distribution too. It is given by the following relation in much of same way as in 3:

$$\langle Q^2 \rangle = \langle \alpha_{LO} | \langle \alpha_{RF} | QQ | \alpha_{RF} \rangle | \alpha_{LO} \rangle \tag{8}$$

In addition we need:

$$\langle Q \rangle^2 = \begin{matrix} 4|\alpha_{LO}\alpha_{RF}|^2 \cos((\omega_{RF} - \omega_{LO})t)^2 + ... \\ ... + 4|\alpha_{LO}|^3 |\alpha_{RF}| \cos((\omega_{RF} - \omega_{LO})t) + ... \\ ... + 4|\alpha_{LO}||\alpha_{RF}|^3 \cos((\omega_{RF} - \omega_{LO})t) + 2|\alpha_{LO}\alpha_{RF}|^2 + ... \\ ... + |\alpha_{LO}|^4 + |\alpha_{RF}|^4 \end{matrix} \tag{9}$$

In time average, the squared mean value becomes:

$$\int_{t=0}^{T} \langle Q \rangle^2 dt = 4|\alpha_{LO}\alpha_{RF}|^2 + |\alpha_{LO}|^4 + |\alpha_{RF}|^4 \tag{10}$$

Taking only the signal part into account one arrives at:

$$\int_{t=0}^{T} \langle S \rangle^2 dt = 4|\alpha_{LO}\alpha_{RF}|^2 + |\alpha_{RF}|^4 \tag{11}$$

Now we are able to calculate the variance of the distribution as:

$$\langle Q \rangle = 2|\alpha_{LO}\alpha_{RF}| \cos((\omega_{RF} - \omega_{LO})t) + |\alpha_{LO}|^2 + |\alpha_{RF}|^2 \tag{12}$$

$$+2(|\alpha_{LO}|^2 + |\alpha_{RF}|^2) < \langle Q^2 \rangle - \langle Q \rangle^2 < +2(|\alpha_{LO}| + |\alpha_{RF}|)^2 \tag{13}$$

From this the signal-to-noise ratio is at first glance obtained as:

$$\frac{\langle Q \rangle^2}{\langle Q^2 \rangle - \langle Q \rangle^2} = \frac{4|\alpha_{LO}\alpha_{RF}|^2 + |\alpha_{LO}|^4 + |\alpha_{RF}|^4}{+2|\alpha_{LO}|^2 + 4|\alpha_{LO}\alpha_{RF}| + 2|\alpha_{RF}|^2} \approx \frac{1}{2}|\alpha_{LO}|^2 \tag{14}$$

Nevertheless, one has to exclude the LO fluctuations from the signal term. A more correct relation is then obtained as:

$$\frac{\langle S \rangle^2}{\langle Q^2 \rangle - \langle Q \rangle^2} = \frac{4|\alpha_{LO}\alpha_{RF}|^2 + |\alpha_{RF}|^4}{+2|\alpha_{LO}|^2 + 4|\alpha_{LO}\alpha_{RF}| + 2|\alpha_{RF}|^2} \approx |\alpha_{RF}|^2 \tag{15}$$

This is a result identical to the derivation in Haus given for a balanced diode mixer.

3. SOLVING THE QUANTUM NOISE FOR ARBITRARY DISTRIBUTED PHOTON PROBABILITY DENSITIES

In the general case it is not acceptable to approximate the photon number density probability function by a simple Poisson distribution. There general relations must be derived based on only a single assumption - the expectation value of the photon counting process yields μ photons. As in the Poisson case, this value is broken down in an integer part n and a fractional part less than unity q. Then the probability density to measure exactly m photons becomes:

$$p_m^S = (1-q)\int_{n\to-\infty}^{+\infty} p(m-n)dm + q\int_{n\to-\infty}^{+\infty} p(m-(n+1))dm \tag{16}$$

using the Kronecker delta δ_x. Calculating the expectation value of this distribution yields μ as shown by inspection, for the variance one obtains:

$$D_\phi^2 = \sum_{m=0}^{\infty}(m-\mu)^2 \cdot p_m = \sqrt{q(1-q)} \tag{17}$$

Obviously **??** indicates the "average measurement error" associated with a measurement on the quantum mechanical system. This error is not dependent on the number of photons measured. It is as a quantization noise in Analog Digital Converters simply related to the value of the least significant bit of the conversion.

From a noise point of view, one has to calculate the maximum value of this variance. The maximum error is thus obtained when the time averaged photon number is exactly a half away from the next integer. Therefore the maximum variance is:

$$\frac{dD_\phi^2}{d\mu} = 1 - 2q = 0 \rightarrow D_{\phi,max}^2 = \frac{1}{2}(1-\frac{1}{2}) = \frac{1}{4} \tag{18}$$

This corresponds to the well-known half quantum noise from Literature. Now we have to invoke ergodicity stating that all statistical moments obtained on a set of measurements on a set of identical quantum system performed at a time point are identical to the time average of measurements performed on a single quantum system within a time interval. Then we are allowed to state the following

A field problem involving a certain photon flow per time unit $\frac{\mu}{\Delta t}$ will show a power flow and its maximum variance given by :

$$P_{RF} = \frac{\mu}{\Delta t}h\nu_{RF}; D_{\phi,max} = \frac{1}{\Delta t}\frac{1}{2}h\nu_{RF} \tag{19}$$

Assuming that the radiation is time correlated with a time being larger than the inverse RF bandwidth of the system $\tau_{corr} > \frac{1}{B}$. Then, the band limited power density involves a correlated photon flow and its variance as given by :

$$P_{RF,B} = \mu h\nu_{RF}B = \langle P\rangle; D_{\phi,max,B} = \frac{1}{2}h\nu_{RF}B = p \tag{20}$$

4. A HEB HEATED BY RF BAND LIMITED ELECTROMAGNETIC RADIATION

Applying the above radiation $\langle P\rangle$ to a resistor results in photon absorption in the resistor therefore diminishing the power flow to subsequent parts in the circuit. Nevertheless the variance p is not affected by this absorption since the variance does not depend on the number of photons involved with the flow. This situation becomes inherently different as soon as this resistor becomes nonlinear (by e.g. heating). Applying two sources of radiation, a strong LO given by $\langle P_{LO}\rangle$ and a weak signal source $\langle P_{RF}\rangle$ to a bolometer results in photon absorption. Using a simple heating model for a bolometer one obtains after suppressing the small second order terms:

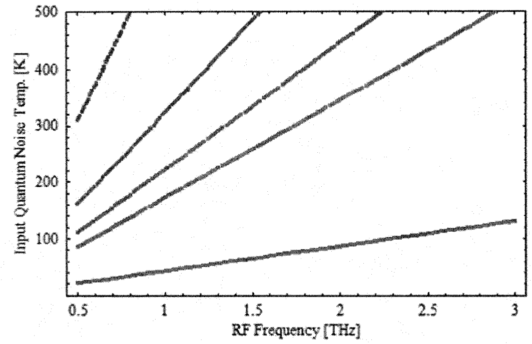

Figure 1. DSB Input Quantum Noise as a function of the RF frequency in the operating point.

$$R(P) = R(\langle P_{LO}+P_{RF}\rangle)+C_{RF}\cdot(\langle P_{LO}\rangle\cdot p_{RF}+\langle P_{RF}\rangle\cdot p_{LO})+C_{DC}P_{DC} = R_0+C_{RF}\cdot(\langle P_{LO}\rangle\cdot\frac{1}{2}h\nu_{RF}B)+C_{DC}P_{DC} \tag{21}$$

This is the way how quantum noise is downconverted in a bolometer. Thus quantum noise follows the thermal fluctuation noise mechanism and becomes later on band-limited within the IF bandwith of the system. It does therefore not deteriorate the noise bandwidth of the system.

Blackbody radiation is caused by a superposition of quantized hollow modes created in a body at a given physical temperature. The time average of the radiation e.g. emitted by the warm optics in front of the receiver (determined by optics losses L_{optics} and the optics temperature T_{optics}) is given by Planck's radiation law found in literature [e.g. Laloé- Tannoudji La mécanique quantique p.280]:

$$\langle P_{Planck,optics}\rangle = L_{optics}\frac{h\nu_{RF}B}{e^{\frac{h\nu_{RF}}{k_B T_{optics}}}-1} \tag{22}$$

In the next step we search for an equivalent thermal noise source at the input to generate the same resistance fluctuation as in 21. This equivalent source is attenuated by optical losses and acts on the whole HEB. In first order, only the fraction heating the hot spot region is able to cause a resistance fluctuation.

$$L_{optics}\cdot\frac{R_0}{R_N}\cdot\frac{h\nu_{RF}B}{e^{\frac{h\nu_{RF}}{k_B T_{QN}}}-1} \approx \frac{1}{2}h\nu_{RF}B \tag{23}$$

Therefore the equivalent noise temperature due to quantum noise becomes:

$$T_{QN} \approx \frac{h\,\nu}{k\,\log(1+2\cdot(1-L_{optics})\cdot\frac{R_0}{R_n})} \tag{24}$$

Please observe that the quantum noise has the same IF bandwidth limitation as the conversion gain. It does therefore not decrease the noise bandwidth of the HEB receiver. The quantum noise occurs at the point where the frequency conversion takes place. Quantum noise is therefore not subject to any optical losses. The "graininess" of the incoming signal is preserved by any losses in front of the bolometer. Referring to output noise powers this is directly obvious. Nevertheless following engineering traditions, noise contributions are usually considered as equivalent input noise temperatures. Consequently the quantum noise at the input increases with increasing optical losses. At frequencies above 2THz, quantum noise dominates the receiver noise.