Characterizing the Full Spatial Optical Behaviour of Power Detectors Using an Interferometric Technique

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Abstract—An experimental method is presented for characterizing the full optical behaviour of few-mode power detectors. The technique involves illuminating the detector with a pair of coherent sources. When the phase of one source is rotated relative to the other, the detector output displays a fringe. Measurement of the complex amplitude of this fringe allows an element of a two-point optical response function to be recovered. By repeating the process for different source positions and orientations, the full two-point response function can be mapped out. From this function it is possible to obtain the full spatial form of each of the modes of the field in which the detector is incoherently sensitive to power, along with the responsivity to the power carried in each of these modes. We describe a scanning system assembled for performing such measurements at frequencies in the range 195 - 270 GHz. Results will be presented for measurements made on a detector that was stopped down to simulate and a few mode planar absorber.

I. INTRODUCTION

Incoherent detectors are used widely in astronomy at THz frequencies, where they provide high sensitivities over large bandwidths. Science requirements in areas such as spaceborne astronomy are driving the need for better optical characterisation of these devices. The characterisation of coherent detectors, such as heterodyne receivers, is a much better understood process. Coherent detectors are responsive to the complex amplitude of a single mode of the incident field. The amplitude, phase and polarisation of this mode can be mapped out using techniques such as near-field scanning [1] or microwave holography [2]. Incoherent detectors, or power detectors as we will refer to them, measure the energy flux of an incident field. In contrast with coherent detectors, a power detector can be independently responsive to the power arriving in several natural optical modes simultaneously. At present, usually only the intensity and polarisation of the overall response is measured. However, this gives an incomplete picture of the behaviour and cannot, for example, be used to calculate how the detector will couple power from an optical system. To fully characterize a device the individual spatial forms of all the natural modes must be determined, along with the responsivity of the detector output to power arriving in each of them.

In this paper we will present the first experimental demonstration of a two-source, interferometric, scheme for characterizing the full optical behaviour of a power detector from power measurements alone. In section II we will outline the formalism developed by Withington [3] and Saklatvala [4] for describing the optical behaviour of power detectors. Here it will be shown that a device’s behaviour can be characterised either in terms of its natural modes or, equivalently, in terms of a two-point dyadic response function. Section III will outline a scheme for recovering elements of this response function using a pair of phase-locked coherent sources. In section IV we will describe a demonstration system for performing measurements of this type in the frequency range 195 - 270 GHz. Finally, in section V we will present data taken at 240 GHz for a TK Instruments Absolute Power Meter, the window of which was stopped down to a 15 mm × 15 mm in order to simulate a few-mode planar absorber.

II. CHARACTERIZING THE OPTICAL BEHAVIOUR OF POWER DETECTOR

The optical behaviour of power detectors has been discussed in detail by Withington [3] and Saklatvala [4]. A summary of their main results will be provided here. Consider a power detector being illuminated by a temporally-stationary field, as shown in figure 1. At each radiation frequency \( \nu \), it is possible to identify a set of modes \( \{ U_n(r, z_0, \nu) \} \) of the electric field over a plane \( z = z_0 \) in which the detector is independently responsive to power. We will refer to this set as the natural optical modes of the detector at \( \nu \). Assume the output of the detector varies as \( \alpha_n(\nu) \) with the square-modulus of the amplitude of the mode in the incident; i.e. the
\( \alpha_n(\nu) \) is proportional the responsivity of the detector output to power arriving in that field mode. Letting \( \mathbf{E}(r, z_0, \nu) \) denote the electric field incident over the plane as a function of the frequency, the detector output \( P \) is given by

\[
P = \int_0^\infty \sum_n \alpha_n(\nu) \left| \int \mathbf{U}_n^*(r, z_0, \nu) \cdot \mathbf{E}(r, z_0, \nu) \, d^2r \right|^2 \, d\nu.
\]

If the incident field is spatially partially-coherent, then the expected value of the output is found by taking the ensemble average \( \langle P \rangle \) of \( P \). By algebraic manipulation, \( \langle P \rangle \) can be put into the dyadic form

\[
\langle P \rangle = \int_0^\infty \sum_n \alpha_n(\nu) \left| \int \mathbf{D}(r_1, r_2, \nu) \cdot \mathbf{E}(r_1, r_2, \nu) \, d^2r_1 \, d^2r_2 \, d\nu, \tag{2}
\]

where the following definitions have been made:

\[
\mathbf{E}(r_1, r_2, \nu) = \langle \mathbf{E}(r, z_0, \nu) \mathbf{E}^*(r, z_0, \nu) \rangle \tag{3}
\]

\[
\mathbf{D}(r_1, r_2, \nu) = \sum_n \alpha_n(\nu) \mathbf{U}_n(r, z_0, \nu) \mathbf{U}_n^*(r, z_0, \nu). \tag{4}
\]

\( \mathbf{E}(r_1, r_2, \nu) \) is the second-order correlation function of the electric field and encodes the state of spatial coherence of the incident field. The second dyadic, \( \mathbf{D}(r_1, r_2, \nu) \), incorporates all the information about the optical behaviour of the detector. We shall refer to it subsequently as the Detector Response Function (DRF). It is the DRF that can be recovered directly by experiment, as will be described in the next section. It follows from \( \mathbf{D} \) that the natural modes of the detector may then be found by a diagonalization procedure. A useful physical picture of the DRF is as the second order correlation function of the state of coherence of the field to which the detector is sensitive. The detector output depends on the amplitude of component of the incident field that projects into this state. This projection operation is described by \( \mathbf{D} \).

Once the DRF has been measured on one plane, its form on another plane can be computed using partially-coherent optics. It is straightforward to show that the DRF propagates through space in the same way as the second order correlations in a partially coherent field \( \mathbf{D} \). This can be done in particularly elegant and computationally efficient way by propagating the natural modes individually, then re-assembling the DRF by superposing the transformed fields together according to \( \mathbf{D} \). Similarly, the full spatial sensitivity of the detector on the sky when it is placed on a focal plane can be found by back-scattering the DRF through the telescope. This is unlikely to be a unitary process, in which case the natural modes of the detector will no longer diagonalize the dyadic response function on the sky. Instead, the transformed response function and the new set of natural modes should be thought of as those of the complete instrument.

III. MEASURING THE DETECTOR RESPONSE FUNCTION

The detector response function at a particular frequency can be measured using a pair of monochromatic sources, phase-locked together at a controllable phase angle \( \Delta \phi \). For clarity of discussion, we will assume each source generates a point-like excitation of the electric field on \( z = z_0 \). However, it is a straightforward to account for the true form of the probe field. The illuminating field is given by

\[
\mathbf{E}(r, z_0, \nu) = |E_a| \hat{p}_a \cdot \mathbf{E}(r_a, \nu_0) + |E_b| \hat{p}_b \cdot \mathbf{E}(r_b, \nu_0) + 2|E_a| |E_b| \hat{p}_a \cdot \mathbf{E}(r_a, \nu_0) \cdot \hat{p}_b. \tag{5}
\]

Substituting (5) into (2), we find that the detector output is

\[
\langle P \rangle = |E_a|^2 \hat{p}_a \cdot \mathbf{D}(r_a, r_a, \nu_0) \cdot \hat{p}_a + |E_b|^2 \hat{p}_b \cdot \mathbf{D}(r_b, r_b, \nu_0) \cdot \hat{p}_b + 2 \text{Re} \left[ E_a^* E_b e^{i\Delta \phi} \hat{p}_a \cdot \mathbf{D}(r_a, r_b, \nu_0) \cdot \hat{p}_b \right]. \tag{6}
\]

To obtain this expression, use has been made of the Hermicity of the DRF, \( \mathbf{D}(r_a, r_b, \nu_0) = \mathbf{D}^\dagger(r_b, r_a, \nu_0) \), which follows from (4), (6) shows that as the phase angle \( \Delta \phi \) between the sources is rotated, the detector output will show a fringe pattern. Provided \( E_a, E_b \) and \( \Delta \phi \) are known, it is possible to infer \( \hat{p}_a \cdot \mathbf{D}(r_a, r_b, \nu_0) \cdot \hat{p}_b \) from the amplitude and phase shift of the fringe. Measurements of \( \hat{p}_a \cdot \mathbf{D}(r_a, r_a, \nu_0) \cdot \hat{p}_a \) and \( \hat{p}_b \cdot \mathbf{D}(r_b, r_b, \nu_0) \cdot \hat{p}_b \) can be made by switching each source off alternately. By repeating the process for different source positions and polarisations, \( \mathbf{D}(r_a, r_b, \nu_0) \) can be mapped out in full.

In most cases it will be possible to reconstruct the full DRF without having to explore all possible source configurations. It is only necessary to take sufficient measurements to account for the number of degrees of freedom in the DRF, as determined numerically by (4) or physically by the degree of spatial coherence. One way of achieving this is to project the DRF into a finite set of appropriate basis functions: for example, a set of Gauss-Hermite modes. In an experiment, the estimates of the projection coefficients could be updated each time a certain amount of new data is taken. Data taking would stop when the estimates have converged and the error is at a desired level.

IV. EXPERIMENTAL SYSTEM

A demonstration system has been assembled for making measurements on power detectors over the frequency range 195 - 270 GHz, which is shown in figure [2]. Figure [3] is a block diagram of the major components of the scanning system. The sources are a pair of rectangular waveguide probes. These are moved independently around the \((x, y)\)-plane by a set of four computer-controlled slides. Both probes...
are driven in the TE01 mode by a pair of phase-locked RF synthesizers operating at 8.1 - 11.25 GHz through separate ×24 multipliers from Virginia Diodes. They are orientated so the resulting electric field is y-polarised. The multipliers are mounted on the slides and are connected to the synthesizers through amplitude- and phase-stable cables to minimize errors from cable flexure. Absolute values of \( E_a \) and \( E_b \) were not measured. Instead, consistent system settings were used to facilitate the comparison of \( \hat{y} \cdot \mathcal{D}(r_a, r_b, \nu_0) \cdot \hat{y} \) between different data sets.

The phase angle between the sources was made to rotate by setting the synthesizers at slightly different frequencies \( f_a \) and \( f_b \). This causes the phase angle between the sources to rotate at a frequency \( \Delta v = 24|f_a - f_b| \), generating a temporal fringe pattern in the detector output at the same frequency (right-hand side of figure 3). A phase reference was provided by superposing the outputs of the synthesizers and then detecting the envelope of the combined signal using an Agilent 8471E power detector (left-hand side of figure 3). A Wilkinson splitter was used as the combiner. The resultant signal has frequency \( \Delta f = |f_a - f_b| \) and its phase is locked to \( \Delta \phi \) at the synthesizer level. Before it can be used as a reference for the phase shift of the detector fringe, the frequency of the reference must be multiplied by a factor of 24.

Characterisation measurements were made on a TK Instruments Absolute Power Meter. This was mounted on a fifth computer-controlled slide, allowing the distance from the probes to be changed. The front window of the detector was stopped down to a 15 mm × 15 mm aperture to simulate a few-mode detector; while the rear window was blanked-off to minimise interference from stray light. The SNR of the fringe measurement was found to be greatest for \( \Delta \nu \approx 8 \) Hz. The frequency of the corresponding phase reference is 0.333Hz, which is too small for the signal frequency to be multiplied up easily electronically. This prevented us from using a lock-in amplifier to measure the complex fringe amplitude. A software based scheme was adopted instead, which involved logging both the detector output and phase reference to computer for each source arrangement. In post-processing, fringe models were fitted to the signals using the Bayesian method from Chapter 4 of [7]. This requires an accurate value for \( \Delta f \), which was derived from the reference channel using a Gaussian-windowed discrete Fourier transform. The fitted phase shift for the reference, \( \phi_r \), and detector output, \( \phi_f \), were then used to find the phase shift of the detector fringe, \( \phi_d = \phi_f - 24 \phi_r \).

The degree of contamination of the data from multi-path reflections within the apparatus was reduced mainly by spatial averaging. Each of the data points in section [V] is an average over the \( z \)-direction, based on 10 measurements at points over a range of \( \pm \lambda \). Each measurement was based on a 20 s fringe recording made at a sampling rate of 1 kHz. The stated errors in the average amplitude and phase are the standard error in the mean of each quantity over the 10 measurements. As an additional measure to reduce contamination, absorber was applied to the detector aperture plate to disrupt the main reflection path.

Measurements of \( E_a^* E_b \hat{y} \cdot \mathcal{D}(r_a, r_b, \nu_0) \cdot \hat{y} \) with \( r_a = r_b \) cannot be made directly, as it is impossible to overlap the probes. Instead, single-source-scans were made with each probe in turn. By taking the square root of the product of these traces, it follows from (6) that a measurement with the correct factor of \( |E_a||E_b| \) can be obtained. The sensitivity of the single-source measurements was improved by inserting switches at A in figure 3 to modulate the probe output at 8 Hz. To account for the effect of the insertion loss of the switch, the data was for each probe was rescaled using an on-axis measurement made using a chopper wheel to modulate the signal.

V. RESULTS

The probes were aligned vertically (\( y \)-direction) with the centre of the aperture on a plane 153 ± 1 mm from the aperture plane. Measurements were then made of the variation of the DRF in the horizontal direction (\( x \)-direction) at 240 GHz. This data is shown in figures 4 and 5. As part of the analysis, we decided to see how well the data could be modelled by assuming the DRF of the detector is uniform, unpolarized and spatially incoherent over the aperture. With these assumptions, the detector output is given by

\[
\langle P \rangle \propto \int |E(r, z, \nu)|^2 \, d^2 r,
\]

where the integral is taken over the aperture. An approximate form for the electric field at the detector plane is

\[
E(r, 0, \nu) = \frac{1}{r_a} E_a \sqrt{g(\theta_a)} |\hat{y} e^{-ik_0 x \sin \theta_a} + \frac{1}{r_b} E_b \sqrt{g(\theta_b)} |\hat{y} e^{-ik_0 x \sin \theta_b} e^{2\pi i|\Delta \nu| t}|
\]

where \( k_0 \) is the angular wavenumber of the radiation and \( r, \theta \) are the radial distance and zenith angle of the probes at the aperture centre. \( E_a \) and \( E_b \) are complex amplitudes and \( g(\theta) \) is the gain pattern of the horn in the plane. By propagating the TE01 mode to the far-field using plane wave methods, it can be shown that

\[
g(\theta) \propto \frac{\sin(\pi w \sin \theta / \lambda)^2}{\lambda^2 |x^2 - (2\pi w \sin \theta / \lambda)^2|^2},
\]

where \( \theta \) is the angle between the source and detector, \( r \) is the distance from the source to the detector, and \( x \) is the distance from the detector to the aperture centre.
where \( \lambda \) is the wavelength of the radiation and \( w \) and \( h \) are the horizontal and vertical dimensions of the probe aperture. Assume the aperture has side length \( p \). Substituting (5) into (7), we find the complex amplitude \( C \) of the detector fringe is expected to have the form
\[
C = 2\alpha p^2 |E_a||E_b| \left[ \frac{g(\theta_a)}{r_a} \right] \left[ \frac{g(\theta_b)}{r_b} \right] \\
\times \exp[i(\Delta \psi - \Delta \nu [k_0(r_a - r_b) + \phi_a - \phi_b]/|\Delta \nu|)] \\
\times \sin \left[ \frac{p}{\lambda} (\sin \theta_a - \sin \theta_b) \right],
\]
(10)

where \( \Delta \nu = \nu_b - \nu_a \); \( \phi_a \) and \( \phi_b \) are the arguments of \( E_a \) and \( E_b \); \( \alpha \) and \( \Delta \psi \) account for the readout electronics. In each plot, the solid lines show a least square fit of this model to the data assuming \( E_a, E_b, \alpha \) and \( \Delta \psi \) as unknowns.

Figures 4a and 4b are plots of the amplitude and phase of the fringe as a function of \( x_a \) with \( x_b = 0 \) mm. The amplitude points have been normalised to the value on-axis \((x_a = x_b = 0 \text{ mm})\) calculated using the model fit. For clarity, the error bars on the amplitude data are not shown. They are on the order of \( \pm 0.01 \) on all points. The phase values have been corrected for the fitted values of \( \phi_a, \phi_b \) and \( \Delta \psi \). Excellent agreement is observed between the model and the data. The fitted value for the separation of the source and detector planes was 152 mm, which is close to the measured value of 153 \( \pm 1 \) mm. Figure 5 shows similar amplitude plots for different values of \( x_b \). All of the plots have been normalised to the experimentally measured fringe amplitude at \( x_a = x_b = 0 \) mm. Each line of best fit was calculated by fitting the model to the corresponding set of data, rather than attempting a global fit. It can be seen from the data that the model fits begin to worsen at high \( x_b \). This may either be a failure in the model for the horn gain, or a failure in the assumption that the response over the aperture is uniform. For example, at high angles we may begin to see the effect of reflections at the internal window in the detector. Additionally, the peaks on the curves fitted to the \( x_b \neq 0 \) data sets should all lie on the envelope function \( x_a = x_b \). The fact they do not is most likely a result of issues with the calibration of the single-source scans.

VI. CONCLUSIONS

We have demonstrated an interferometric technique for characterizing the full optical behaviour of multi-mode power detectors. The detector under test is illuminated with a pair of phase-locked, monochromatic, sources. When the phase angle between the sources is rotated, the detector output traces out a fringe pattern. From the complex amplitude of the fringe we can determine the value of the Detector Response Function (DRF) at the source positions. By repeating the procedure for different source positions, the DRF can be mapped out in full. Measurements of both the fringe amplitude and phase were made on a stopped down detector at 240 GHz. The data collected was found to be well described by model that assumes the DRF is spatially incoherent over the aperture.

The basic experimental scheme described here is applicable at all wavelengths with suitable modification. For example, a measurement at optical wavelengths might use laser-fibre probes and micrometer stages in place of slides. Other realizations of the same experiment are also possible. Much of the current design was influenced by the need to make measurements on a slow, room-temperature, detector. Cryogenic astronomical detectors are much faster and have much greater sensitivity, so a simplified read-out scheme could be adopted.

Although the emphasis in this paper has been on detector characterisation, we also envisage the scheme having application in other fields of research. A near-field version of the scheme might, for example, be useful for measuring non-local conduction effects and microstructure in thin conducting films.
Fig. 5. Fringe amplitude (a) and phase (b) as a function of the $x$-coordinate of source A with source B in different positions. The fringe amplitude is normalised to the value at $x_a = x_b = 0$ mm measured using a chopper wheel and the single sources. The phase shifts $\phi_a$, $\phi_b$ and $\Delta\psi$ have been subtracted off the phase data. The solid line shows the model fit to each data set. Radiation frequency = 240 GHz.

REFERENCES