Bandwidth of a 4.7 THz asymmetric Fourier grating

Y. Gan, B. Mirzaei, J. R. G. Silva, W. Laauwen, F.F.S. van der Tak, and J. R. Gao

***Abstract*—We present an analysis of the bandwidth of a preliminary designed asymmetric 8-pixel Fourier grating as the beam multiplexer for the 4.7 THz local oscillator of the GUSTO mission. We take the GUSTO grating as an example to address the bandwidth question although GUSTO itself does not need to operate over a wide frequency range. By illuminating single beams with different frequencies from 4.445 THz to 5.045 THz to the grating, we simulated the changes in the grating's performance in three aspects using COMSOL Multiphysics: diffraction efficiency, power distribution, and the angular distribution of the output beams. These parameters can reduce the coupling efficiency between the output beams of the grating and the beams of the mixer array of GUSTO. The grating's bandwidth is calculated to be 250 GHz, which is sufficient for many applications.**

***Index Terms*— Fourier grating, Bandwidth, Mixer array, Coupling efficiency.**

I. INTRODUCTION

HETERODYNE detection is widely used to detect atomic fine structure lines and molecular rotational lines in the terahertz (THz) frequency region from the interstellar medium (ISM). This technique provides very high spectral resolution, R > 10^6. Heterodyne receivers convert a sky signal in THz down to gigahertz frequency by mixing the weak celestial signal with a signal from a local oscillator (LO). In the supra-THz region (> 1 THz), quantum cascade lasers (QCLs) provide considerably higher output powers compared to LO sources based on multipliers [1]. Therefore, the 4.745 THz band in the Galactic/ Extragalactic ULDB Spectroscopic Terahertz Observatory (GUSTO) [2], aiming for detection of [OI] line emission from the Milky Way and nearby galaxies, especially the Magellanic Clouds, combines a QCL and a Fourier grating as the LO for the 8-pixel hot electron bolometer (HEB) lens-antenna coupled mixer array. The array is crucial to enhance observation speed. In order to detect the Doppler shift caused by the linear velocity of the ISM beyond our galaxy, a sizable tuning range of LO will be necessary since the IF bandwidth of a HEB mixer available in this frequency range is limited. So, an interesting question is how large the bandwidth of a Fourier grating can be.

A Fourier grating as a multiplexer to diffract a single beam from the QCL to multiple beams is designed to work across a finite frequency range. For a given Fourier grating, the change of the source frequency causes a loss in the coupling efficiency between the image beams of the grating and the beams of the mixers [3]. So the bandwidth of a Fourier grating is characterized by the interplay between the grating and an array. Thus, the bandwidth analysis should take the specific array used into account. Fig. 1 shows a conceptual diagram of the 4.745 THz array receiver . A single QCL beam is first diffracted to 8 beams by a reflective Fourier grating, which are collimated by a parabolic mirror, becoming parallel to each other. These 8 parallel beams are then coupled to the lens-antennas of the mixer array. We analyze the bandwidth of an 8-pixel Fourier grating designed for the 4.745 THz band of GUSTO, as an example. We notice that GUSTO itself does not require a large bandwidth since it aims to detect [OI] lines from the Milky Way. Furthermore, we took a GUSTO grating design available at the time when we performed the analysis, which is not the final one. We do believe that the approach we present here should be applicable for any combinations of a grating LO with an array.

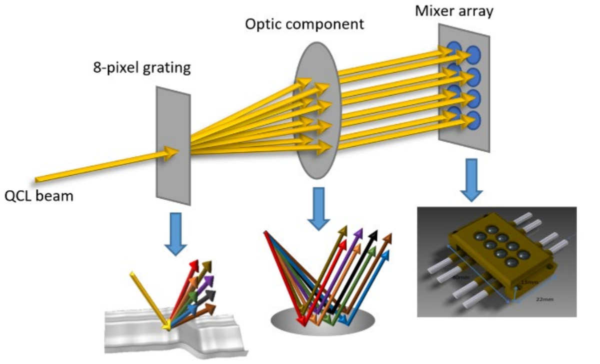

Fig.1. Conceptual diagram of the 4.745 THz 8 beam local oscillator for the GUSTO mission. The QCL beam is first diffracted by a

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This work was supported in part by EU RadioNet Project under Grant 730562 and in part by TU Delft Space Institute. Y. Gan is supported by China Scholarship Council Studentship (201706030153) and by University of Groningen.

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reflective phase grating, and is then collimated by a parabolic mirror to make the 8 parallel beams. These 8 beams are coupled to a quasi-optical mixer array.

II. SIMULATED 4.745 THZ FOURIER GRATING

A phase grating consists of a periodic structure to diffract a single beam to multi-beams in different directions through phase modulation. According to diffraction theory, the diffracted far field distribution from a grating can be expressed as the Fourier transform of the grating's transmission/reflection function [4]. Gratings using Fourier synthesis technique to achieve continuous phase-only groove shapes are called Fourier gratings [5]. To design a Fourier grating, we expanded the phase modulation function of the grating to Fourier series with a set of Fourier coefficients a_n. Using the Fast Fourier Transform and the Standard multidimensional minimization algorithm in Matlab, we found a set of a_n for a one-dimensional grating with the desired number of pixels. A two dimensional grating is generated by superimposing two 1D profiles orthogonally. In our case, one direction is a 2-pixel 1D grating, and the other direction is a 4-pixel 1D grating. According to the grating equation $D(sin\theta_m - sin\theta_i) = m \cdot \lambda$, the direction of the output beam θ_m in the diffraction order m (integer) is determined by the grating period D, the incident angle with respect to the normal of the grating θ_i and working wavelength λ. An asymmetric grating is designed to accommodate the requirements of the GUSTO optical system by employing consecutive diffraction orders [6]. For the 4-pixel 1D grating, we chose the (-2,-1,0,+1) diffraction orders, and for the 2-pixel 1D grating, we chose the (-1,0) diffraction orders. The surface topology and specifications (as simulated with COMSOL Multiphysics [7]) of the design are shown in Fig. 2 and Table I, respectively.

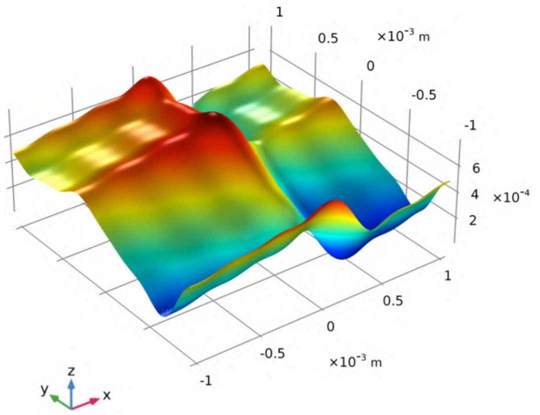
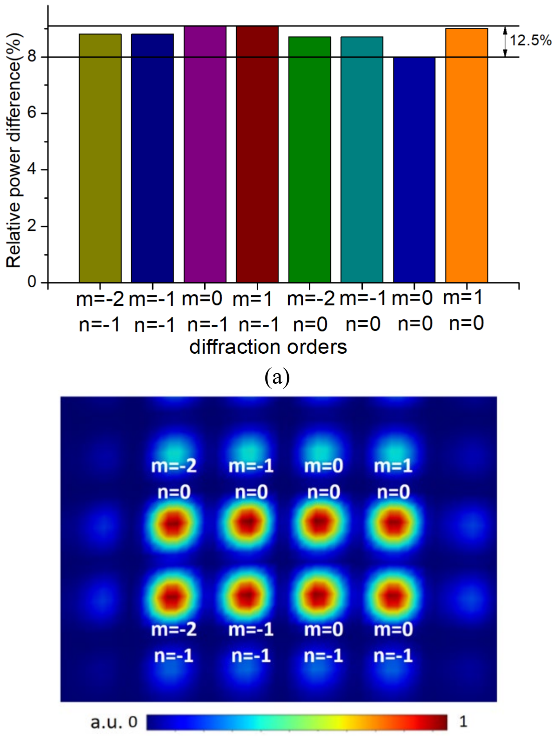


Fig. 2. The surface topology of one unit cell of the grating [6]. The input QCL beam illuminates the grating surface with an incident angle 15° in the x direction and 0° in the y direction.

TABLE I
Grating Specifications

Working frequency	4.745THz
Material	Aluminum
Angular distribution	1.83°
Incident angle	15°
Unit cell size	2.04mm×1.979mm
Diffraction orders	(-2,-1,0,1)(-1,0)
Diffraction efficiency	70.2%
Uniformity deviation = $(I_{max}-I_{min})/I_{average}$	12.5%

If the incident angle increases, the power variation among the output beams becomes larger [8]. The different unit cell size in two directions is to make the angular distribution in these two directions the same. The diffraction efficiency, defined as the ratio of the total diffracted beam power to the power of the incoming beam. The power variation is simulated by using COMSOL Multiphysics and by importing the surface topology of the grating. We apply the periodic port with periodic boundary condition in the RF module, and extract the S-parameters of the port. The power distribution of the output beams is plotted in Fig. 3(a), where we find the largest variation to be 12.5% (between the maximum power and the minimum power). By importing the surface profile of the designed unit cell of the grating and by repeating it in both orthogonal directions, while taking the input as a Gaussian beam, we simulated the far field beam pattern of the grating. The outcome is shown in Fig. 3(b), where the m and n are the diffraction orders in both directions. From the results in Fig. 3, we conclude that the grating achieves a good power uniformity among the output beams.



(a)

(b)

Fig. 3. (a) Power distribution of the 8 output beams from the grating. The largest variation is 12.5% (between the maximum power and the minimum power). (b) Far field beam pattern of the grating. The m and n are the diffraction orders in two directions.

III. CALCULATION OF THE GRATING BANDWIDTH

The 8 output beams from the grating are used to pump an 8-pixel mixer array in the 4.745 THz band of GUSTO. The power variation among the LO beams can degrade the sensitivity of the mixers in the array since it depends on LO power. We assume that the latter should be within 5% of the optimal value, and we also assume all HEBs in the array require the exact same LO power. The corresponding LO power variation is estimated to be ~ 21% using the isothermal technique [9,10,11]. *In this paper, we use this criterion to define the bandwidth of the grating, namely, as the frequency range changes such that the LO power variation among the array mixers is within 21%.*

The change of the frequency affects the performance of the grating in three aspects: (a) frequency change causes a change in the diffraction efficiency η; (b) Frequency change causes a change in the power distribution of the outcome beams; (c) According to the grating equation, the frequency change affects also the angular distribution of the output beams, which reduces optical coupling to the lens-antennas of the mixer array.

Based on these three aspects, we defined Gaussian beams with different frequencies from 4.445 THz to 5.045 THz to illuminate the grating used in COMSOL Multiphysics. The simulation results, in which the changes in diffraction efficiency when the frequency of the beams is changed, are shown in Table II.

TABLE II

The diffraction efficiencies η corresponding to different frequencies f from 4.445THz to 5.045 THz

$f(THz)$	**4.445**	**4.545**	**4.645**	**4.745**
η	70.6%	70.5%	70.4%	70.2%
$f(THz)$	**4.845**	**4.845**	**5.045**	
η	70.0%	69.7%	69.4%	

Table II suggests that the maximal change in diffraction efficiency by varying the frequency is 0.8%, which is negligible compared to other effects (the power distribution and the angular distribution).

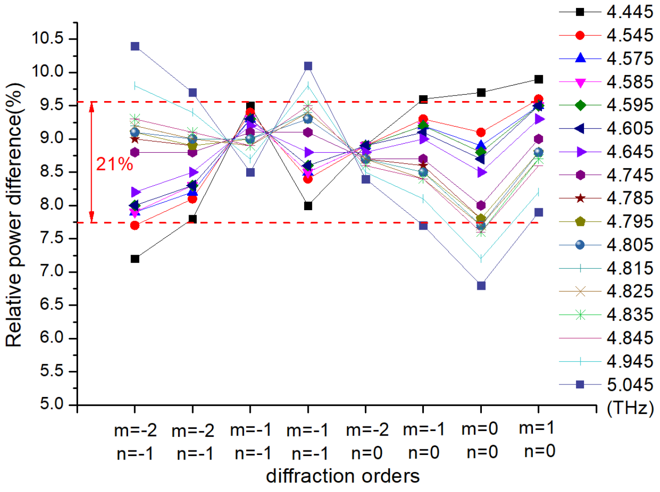


Fig. 4. Power distribution of the output beams when the working frequency changes from 4.445 THz to 5.045 THz. Red dashed lines indicate the LO power boundaries, within which the powers of the output beams vary within 21% around their average value.

When the frequency changes, the power distribution among the output beams varies. When the grating works at the nominal (or designed) frequency of 4.745 THz, its maximal power variation of the output beams is 12.5% (between the maximum power and the minimum power). When the frequency changes, this number becomes larger. Fig. 4 plots the power distribution of the output beams of the grating operated at different frequencies. The red dashed lines indicate the LO power boundaries, within which the power of the output beams varies within 21% around their average value. From Fig. 4, when the frequency changes between 4.575 THz and 4.825 THz, the relative powers of all the output beams from the grating vary within 21% around their average value (8.6%). Base on this we derive that the bandwidth of the grating is 250 GHz.

Now we examine the angular distribution of the output beams as a function of the frequency. Since the pixel spacing of the mixer array is fixed, the change in the spacing of the beams can lead to offsets with respect to those of the mixer array. According to the relation between the offset and Gaussian beam coupling [12], we can calculate the coupling loss caused by the offset at different frequencies. The results are shown in Fig. 5.

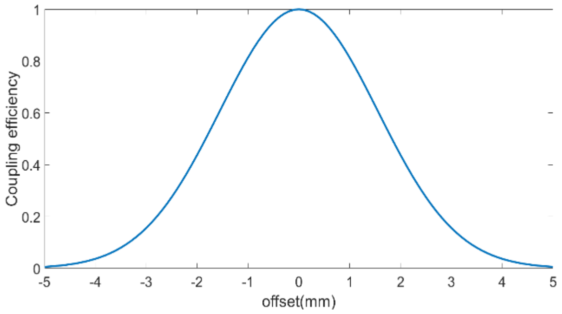


Fig. 5. The change of the coupling efficiency between the output beams and the mixer array caused by the spatial offset between them (or called misaligned).

Based on the grating equation, we calculated the angular distribution of the output beams and the offset between the output beams and the lens-antennas at different frequencies. Then, taking advantage of the offset dependence of coupling efficiency in Fig. 5, we obtained the coupling loss caused by the offset. The next step is to calculate the distribution of powers that are effectively coupled to the mixer array. The results are shown in Fig. 6.

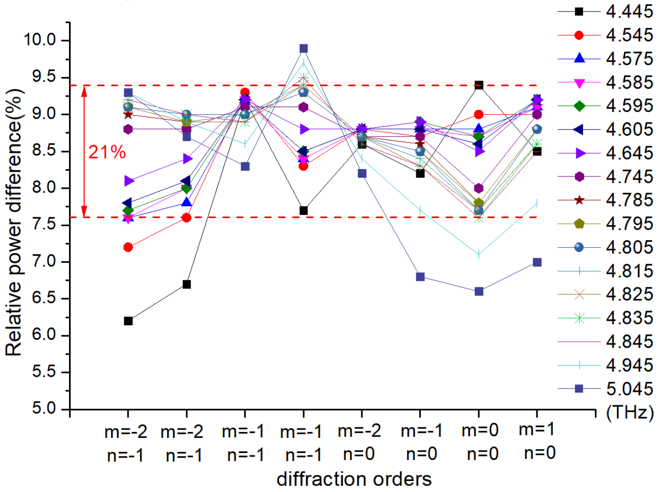


Fig. 6. Power distribution of the output beams, taking the angular distribution into consideration when the working frequency changes from 4.445 THz to 5.045 THz. Red dashed lines indicate the LO power boundaries, within which the powers of the output beams vary within 21% around their average value.

In Fig.6 the red dashed lines indicate the LO power boundaries, within which the powers of the output beams vary within 21%, where the average value is 8.5 %. We find that the coupled powers are still within 21% with frequency changing between 4.575 THz and 4.825 THz. Therefore, from this aspect, the bandwidth of the grating is also 250 GHz.

IV. CONCLUSION

We analysed the bandwidth of an asymmetric grating preliminary designed for the 4.745 THz local oscillator for GUSTO in three aspects; diffraction efficiency, power distribution and angular distribution. We found that for a 4.745 THz asymmetric grating, its output beam power variation remains within 21% when the operating frequency changes from 4.575 THz to 4.825THz, which gives a 250 GHz frequency bandwidth. The effect is dominated by the power distribution and angular distribution, while the diffraction efficiency remains nearly unchanged. The 250 GHz bandwidth corresponds to 15,000 km/s linear velocity, which is much more than the line widths of the galactic (~100 km/s) and extragalactic (~1000 km/s) objects.

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