On the Comparison Between Low Noise Amplifiers and Photonic Upconverters for Millimeter and Terahertz Radiometry

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***Abstract*—We analyze the feasibility of upconverting THz radiation to the optical domain for high sensitivity room temperature radiometry, as an alternative to radiometers based on low noise amplifiers (LNAs). Following a semiclassical approach, the noise performance of the upconverters is studied. A similar analysis is followed to model the thermal and quantum sources of noise in low noise amplifiers. A comparison between both schemes is done, showing the potential of the upconversion approach if high efficiencies are accomplished. This is due to the low thermal occupation achievable in whispering-gallery mode upconverters and the fact that the minimum introduced noise is not fundamentally bounded by a quantum limit when direct detection is performed.**

***Index Terms*—WGM, upconverters, radiometers.**

I. Photonic upconversion for THz radiometry

Many applications of technological and scientific interest require the accurate detection and measurement of the electromagnetic power radiated by sources of thermal nature. It is in general desired to retrieve the temperature of the source from such power measurements that are performed with radiometers. Radiometers collect the thermal radiation with antennas and then a receiver stage measures the average power by integrating over an interval τ. As will be shown in the following section, even in the ideal assumption that the radiometer does not introduce noise, the randomness of the source's instantaneous intensity and photon arrival unavoidably leads to an uncertainty in the power measurement that decreases with the observation time τ. Non-ideal mechanisms present in a real radiometer such as impedance mismatchings, dissipation losses and internally generated thermal noise worsen the uncertainty of the measurements done by the instrument.

Conventional high sensitivity radiometers consist of a power detector whose input is pre-amplified by a low noise amplifier (LNA) which contributes significantly to the overall noise of the instrument. While cryogenic LNAs exhibit much lower noise temperatures than room temperature ones, their performance is severely degraded at high frequencies in the millimeter and sub-millimeter wave range. Indeed, it has been suggested that there is a limit in the minimum noise temperature of field effect transistors (FETs) in general. For indium phosphide (InP) high-electron-mobility-transistors (HEMTs) this limit is about 4.5 times the quantum limit [1], which is close to the state-of-the-art. The quantum limit is the minimum noise temperature that any amplifier can exhibit and is given (in Rayleigh-Jeans units) by $T_e = h\nu_0/k_B$ in the high gain limit, where ν_0 is the operation frequency, and h and k_B the Planck and Boltzmann's constants respectively. The existence of such limit has a fundamental origin and comes from the Heisenberg's uncertainty principle: if amplification occurred without noise, the output of the amplifier could be measured with an uncertainty in energy and time lower than the minimum enforced by Heisenberg's uncertainty's principle. This is a consequence of the fact, that the number of photons at the output is higher than the number of photons at the input.

A different approach potentially useful for high sensitivity radiometry is the nonlinear upconversion of the thermal THz radiation to the optical domain, and its subsequent detection with non-cooled photodetectors. Under certain conditions the upconversion process is intrinsically noiseless as no photon multiplication occurs [2]. Indeed, the number of photons at the output and at the input of an ideal upconverter matches, corresponding to a unity photon conversion efficiency $\eta = 1$. In this case, even though there is power amplification since photons are more energetic at the output than at the input, Heisenberg's uncertainty principle holds at the output with no need of added noise. Therefore, the insertion of an ideal upconverter does not worsen the signal to noise ratio of the input, in contrast to the insertion of an ideal LNA. Even though not fundamentally limited, a real upconverter introduces noise since it exhibits a non-ideal efficiency $\eta < 1$ and is thermally occupied due to its physical temperature above $0\,\mathrm{K}$. Nevertheless, efficient upconverters can be designed with resonant structures made of low absorption nonlinear crystals that reduce significantly the upconverted thermal noise at room temperature [3]–[6]. These facts justify the study of nonlinear upconverters for potential high sensitivity detection in the millimeter and submillimeter wave range with less stringent cooling requirements than LNAs.

II. Radiometer equation in upconverters

We model an upconverter with $\eta < 1$ as an ideal upconverter whose input is passed through a beamsplitter with coupling coefficient η. Some thermal radiation generated inside the upconverter due to its physical temperature T_p is converted to the optical domain along with the antenna signal. This

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is modeled by means of an artificial input thermal source at temperature T_{eff} superimposed to the antenna signal. In upconverters based on whispering-gallery mode resonators (WGM) for most cases $T_{\text{eff}} < T_p$ provided that the resonator is sufficiently overcoupled to the antenna [3]. The parameters, η and T_{eff} along with the upconversion frequency ν_0 and bandwidth $\Delta\nu$ characterize the upconverter. The bandwidth is defined as

$$\Delta\nu = \frac{\int_0^\infty H(\nu)\,\mathrm{d}\nu}{H(\nu_0)} \tag{1}$$

where $H(\nu)$ is the one-sided power transfer function of the upconverter, such that the total thermal power received by the upconverter can be written as $P = k_B T \Delta\nu$, with T the temperature of the observed source in Rayleigh-Jeans units. This implicitly assumes $H(\nu)$ is sufficiently narrowband to consider the power spectral density of the thermal radiation constant over the transfer function. Similarly, the noise equivalent bandwidth B is defined as [7]

$$B = \frac{\left[\int_0^\infty H(\nu)\,\mathrm{d}\nu\right]^2}{\int_0^\infty H^2(\nu)\,\mathrm{d}\nu} \tag{2}$$

For typical filter shapes, the relation $a = B/\Delta\nu$ is a constant on the order of unity. Similarly, B and $\Delta\nu$ are commonly on the same order of the full width half power bandwidth.

A. Direct detection: Semiclassical radiometer equation

We assume the filter function has a Lorentzian shape $H(\nu) = \frac{\gamma/2\pi}{(\nu-\nu_0)^2+(\gamma/2\pi)^2}$ since this is the case for ultra high-Q WGM upconverters. Thermal radiation passed through $H(\nu)$ is equivalent to Gaussian-Lorentzian chaotic light with coherence time $\tau_c = 1/\gamma = 1/(2\Delta\nu) = 1/B$ whose photodetection statistical properties can be obtained from Mandel's formula [8]. A noiseless photon counter with quantum efficiency $\eta_q \leq 1$ directly detecting the Lorentzian-filtered thermal radiation at temperature $T_A = \langle P_A\rangle/(k_B\Delta\nu)$ where $\langle P_A\rangle$ is the average received power, will count on average $\langle m\rangle = \frac{\tau}{h\nu_0}\eta_q\langle P_A\rangle$ photons during the interval τ. For arbitrary observation time τ, the variance in the photon counts is given by [8]:

$$\operatorname{var}(m) = \langle m\rangle + \langle m\rangle^2\left(\frac{\tau_c^2}{2\tau^2}\right)\left[e^{-\frac{2\tau}{\tau_c}} - 1 + \frac{2\tau}{\tau_c}\right] \tag{3}$$

Rewriting the variance in photon counts as a variance in measured power P_A during $\tau \gg \tau_c$, yields a semiclassical radiometer equation:

$$\operatorname{var}(P_A) = \frac{\langle P_A\rangle^2}{B\tau}\left(1 + \frac{h\nu_0 B}{\eta_q\langle P_A\rangle}\right) \tag{4}$$

When $\eta_q = 1$, Eq. (4) gives the minimum measurement uncertainty of thermal radiation achievable by any detection scheme [9], [10]. It the converges to the classical radiometer equation in the large photon number limit when $\eta_q k_B T_A \gg h\nu_0$ and photon shot noise is negligible compared to excess noise. One of the consequences of non-negligible photon shot noise, is that the signal to noise ratio $\langle P_A\rangle^2/\operatorname{var}(P_A)$ is not independent from the input power.

B. Detection with an upconverter

In a real upconverter, some thermal radiation due to the ambient temperature couples to it. This is accounted by a thermal source at temperature T_{eff}, added to the antenna temperature T_A. Due to the Gaussian and additive nature of thermal radiation and the fact that both sources are uncorrelated, it is expected this superposition of thermal radiation has the same statistics of a single thermal source whose temperature is $T_A + T_{\text{eff}}$. This photon stream is passed through a lossless beamsplitter with photon number transmission η to account for the non-ideal efficiency, and then to an ideal upconverter. The output is then measured by an optical photon counter with quantum efficiency η_q. The ideal upconverter after the beamsplitter does not change the photon statistics but only boosts their energy. Detecting with a noiseless photodetector with quantum efficiency η_q is equivalent to detecting with an ideal photodetector after passing the optical signal through a beamsplitter with photon number transmission η_q which results in the replacement of η by $\eta\eta_q$. The input mean photon number is given by $\langle m_i\rangle = \frac{\tau}{h\nu_0}\langle P_T\rangle$ where $\langle P_T\rangle = \langle P_A\rangle + \langle P_{\text{eff}}\rangle$ and $\langle P_{\text{eff}}\rangle$ is the average power of the artificial source accounting for coupled thermal noise inside the upconverter. On the other hand, the input variance results $\operatorname{var}(m_i) = \langle m_i\rangle + \langle m_i\rangle^2/(B\tau)$ for $\tau \gg 1/B$ (see Eq. (3)). This results in a measured photon mean $\langle m\rangle = \eta\eta_q\langle m_i\rangle$ with variance

$$\operatorname{var}(m) = \eta\eta_q\langle m_i\rangle + (\eta\eta_q)^2\frac{\langle m_i\rangle^2}{B\tau} \tag{5}$$

Rewriting Eq. (5) in terms of the measured incoming power knowing that $\operatorname{var}(m) = \left(\eta\eta_q\frac{\tau}{h\nu_0}\right)^2\operatorname{var}(P_T)$, we have

$$\operatorname{var}(P_T) = \frac{\langle P_T\rangle^2}{B\tau}\left(1 + \frac{h\nu_0 B}{\eta\eta_q\langle P_T\rangle}\right) \tag{6}$$

Hence, the power estimation of the source of interest P_A can be retrieved from the measurement of m photons during τ, as $P_A = \frac{h\nu_0}{\tau\eta\eta_q}m - \langle P_{\text{eff}}\rangle$, assuming $\langle P_{\text{eff}}\rangle$ is a known offset that can be removed. This way, the expected value of the measurement is exactly the mean power of the source $\langle P_A\rangle$ and its variance

$$\operatorname{var}(P_A) = \frac{(\langle P_A\rangle + \langle P_{\text{eff}}\rangle)^2}{B\tau}\left[1 + \frac{h\nu_0 B}{\eta\eta_q(\langle P_A\rangle + \langle P_{\text{eff}}\rangle)}\right] \tag{7}$$

The photon shot noise term in Eq. (7) might be significant for sufficiently high frequencies and low efficiencies $\eta\eta_q$.

III. Comparison with low noise amplifiers

A similar approach can be followed to estimate the variance in the power measured by an LNA based radiometer. Normally, LNAs are only characterized by an equivalent noise temperature referred to its input T_e which is superimposed to the signal to account for all intrinsic LNA noise. This is analogous to T_{eff} in the upconverter. T_e is commonly measured by means of the Y factor method. Then, the radiometric variability is assumed to follow the classical radiometer equation $\operatorname{var}(P_A)B\tau =$

$(\langle P_A\rangle + \langle P_e\rangle)^2$ where $\langle P_e\rangle = k_B T_e \Delta\nu$. It is not clear whether this assumption still holds for LNAs in the millimeter and sub-millimeter wave range when observing cold sources such as the cosmic microwave background with cryogenic LNAs. The reason is that according to Eq. (4), photon shot noise might be significant and the classical radiometer equation is not a good approximation. Therefore, it can be important to quantify the losses introduced by the LNA, which like the upconverter's efficiency, would contribute to the photon shot noise term. These losses are not immediately available since they are masked by the gain of the amplifier in standard measurement setups.

We account for internal thermal noise in an LNA as originating from the ohmic dissipation losses α of the circuits prior to amplification. This is modeled through a beamsplitter whose inputs are the antenna signal and a thermal source at the physical temperature of the LNA T_p, with transmission coefficient α and $1-\alpha$ respectively. Other photon losses which do not reciprocally lead to coupled thermal noise from the ambient are accounted by a beamsplitter with coupling coefficient η_i. In an LNA such losses can be due to an impedance mismatching where part of the incoming power is reflected and radiated back through the antenna. The resulting power feeds an ideal LNA, which still has an intrinsic noise source at the quantum limit level, due to the amplification of the zero point fluctuations [11]. Fundamentally, the minimum noise power at the output of an ideal amplifier of gain G is on average $h\nu_0\Delta\nu\,(G-1)$ [11], which referred to the input, corresponds to $\langle P_{\mathrm{ASE}}\rangle = h\nu_0\Delta\nu\frac{G-1}{G}$, that is, $1-1/G$ photons per second per Hertz of bandwidth and has thermal statistics [11], [12].

Finally, the output of the ideal LNA feeds a power detector (photon counter) with quantum efficiency η_q. The noise contribution of the photodetector will be negligible as long as the amplifier gain G is large enough. However, η_q can be taken into account.

The total mean power received by the ideal amplifier is

$$\langle P_i\rangle = \alpha\eta_i\langle P_A\rangle + (1-\alpha)\langle P_p\rangle + \langle P_{\mathrm{ASE}}\rangle \tag{8}$$

where $\langle P_p\rangle = k_B T_p\Delta\nu$ is the mean power of thermal noise due to the physical temperature of the amplifier. Then, the average output power detected is

$$\langle P_o\rangle = \eta_q G\langle P_i\rangle \tag{9}$$

and its variance for $\tau \gg \tau_c$

$$\mathrm{var}\,(P_o) = \frac{\eta_q^2 G^2\langle P_i\rangle^2}{B\tau}\left[1 + \frac{h\nu_0 B}{\eta_q G\langle P_i\rangle}\right] \tag{10}$$

Therefore, for a given power measurement P_o, the temperature of the antenna can be estimated as

$$P_A = \frac{P_o}{\eta_q G\alpha\eta_i} - \left(\frac{1-\alpha}{\alpha\eta_i}\right)\langle P_p\rangle - \frac{1}{\alpha\eta_i}\langle P_{\mathrm{ASE}}\rangle, \tag{11}$$

assuming the offset $\langle P_e\rangle = \left(\frac{1-\alpha}{\alpha\eta_i}\right)\langle P_p\rangle + \frac{1}{\alpha\eta_i}\langle P_{\mathrm{ASE}}\rangle$ is known and can be removed and the total gain $G_t = \eta_q G\alpha\eta_i$ is also known. The resulting variance in the measured power received by the antenna P_A is given by

$$\mathrm{var}\,(P_A) = \frac{(\langle P_A\rangle + \langle P_e\rangle)^2}{B\tau}\left[1 + \frac{h\nu_0 B}{G_t(\langle P_A\rangle + \langle P_e\rangle)}\right] \tag{12}$$

Equation (12) is analogous to Eq. (7) for the upconverter. In that case the parameters $\eta\eta_q$ and T_{eff} can be known from theoretical models and verified experimentally [3]. For the LNA, the parameters G_t and $\langle P_e\rangle$ can be determined experimentally via the Y factor method. Indeed, by using hot and cold calibrated loads at temperatures $T_h = \langle P_h\rangle/(k_B\Delta\nu)$ and $T_c = \langle P_c\rangle/(k_B\Delta\nu)$ respectively, the ratio Y between measured output mean powers in each case is calculated:

$$Y = \frac{P_{o(h)}}{P_{o(c)}} \approx \frac{\eta_q G\alpha\eta_i(\langle P_h\rangle + \langle P_e\rangle)}{\eta_q G\alpha\eta_i(\langle P_c\rangle + \langle P_e\rangle)} \tag{13}$$

from which $\langle P_e\rangle$ can be obtained as

$$\langle P_e\rangle \approx \frac{\langle P_h\rangle - Y\langle P_c\rangle}{Y-1} \tag{14}$$

The approximations in (13) and (14) are better for longer observation times τ. Similarly, the total gain of the amplifier can be obtained experimentally from

$$G_t \approx \frac{P_{o(h)} - P_{o(c)}}{\langle P_h\rangle - \langle P_c\rangle} \tag{15}$$

Hence, the variance in the antenna temperature estimation of an LNA based radiometer, follows the semiclassical radiometer equation of (4), where besides the antenna temperature T_A, the system temperature T_e as measured with the conventional Y factor method must be included. The photon shot noise factor is signal dependent, but in principle can be made arbitrarily small for sufficiently high amplifier gain G that surpasses the overall losses in the LNA. It is worth noting that any additional gain introduced after the photodetector does not affect the validity of Eq. (12) (as long as the additional introduced noise is negligible). However, such additional gain cannot be included in the definition of G_t, so the experimental determination of G_t via Eq. (15) is only valid when measurements are done right after the power detector. Otherwise, Eq. (15) must be divided by the post-detection gain.

IV. Discussion

Since the effective thermal noise temperature of a WGM-based upconverter is in principle lower than the physical temperature of the resonator $T_{\mathrm{eff}} \leq T_p$ [3], its noise is mainly determined by the low photon conversion efficiencies achieved so far. This is evidenced in the inverse proportionality to the efficiency of the shot noise term in Eq. (7). The simplified LNA model presented in this work considers the sources of noise as being of thermal and quantum origin. The conclusion is that from conventional Y factor measurements, the variance at the output of the LNA can be estimated by means of the semiclassical radiometer equation. It was not clear whether the internal losses and mismatches of the LNA which are not characterized might lead to a significant photon shot noise contribution for the output variance, invalidating the use of the classical radiometer equation. Our theoretical result of Eq. (12) shows that this is not the case provided that the amplifier gain is large enough. This might seem counter intuitive for the following: The penalty carried by amplification is the amplified spontaneous emission (ASE) noise which practically does not change for gains ranging from moderate values $G \approx 10$ to $G \to \infty$. Superficially this can lead to the erroneous conclusion that strong photon loss in an LNA with arbitrarily large gain does not increase the photon shot noise effect in contrast to an upconverter which does not exhibit gain. This is a fallacy, since even though ASE noise is constant, its contribution to $\langle P_e \rangle$ is inversely proportional to the overall loss coefficient $\alpha\eta_i$. In fact, since the term $\langle P_e \rangle$ is quadratic in Eq. (12), whereas the $1/\eta\eta_q$ dependence of photon shot noise term in Eq. (7) is linear, photon loss fundamentally has a stronger negative effect in LNAs than in upconverters.

In order to show the potential of the upconversion approach for submillimeter wave and THz radiometry, we can compare the normalized radiometer variability $\sqrt{\text{var}\left(P_A\right) B\tau}/(k_B\Delta\nu)$ of the upconverter with that of state-of-the-art low noise amplifiers. In the latter case $\langle P_e \rangle = k_B T_e \Delta\nu$ where T_e is the system temperature of the LNA referred to the input in Rayleigh-Jeans units, obtained from Y-factor measurements. We assume the best case for the LNAs when $G \to \infty$. Figure 1 shows these results for some millimeter and submillimeter wave LNAs reported in the literature, and compared with those achievable by an upconverter with $\eta = 10^{-2}$. We have assumed $T_{\text{eff}} = 290\,\text{K}$ for the upconverter as it has been shown that the WGM resonator can always be overcoupled such that T_{eff} is below the physical temperature of the crystal, provided that sufficiently high intrinsic microwave $Q \gtrapprox 20$ is realized with low azimuthal mode order $n \approx 4$ [3]. We also plot the upconverter results for $T_e = 100\,\text{K}$, which is achievable by overcoupling if $Q > 50$ [3]. In lithium niobate, intrinsic Q factors ranging from 400 to 40 are achievable from $100\,\text{GHz}$ up to $2\,\text{THz}$ [13]. The higher the intrinsic Q, the lower the T_{eff} that can be realized via overcoupling [3].

It can be seen from Figure 1 how the theoretically expected sensitivity of the upconversion approach significantly surpasses state of the art HEMT LNAs in the submillimeter range. Indeed, room temperature overcoupled WGM resonators leading to $T_{\text{eff}} \leq 100\,\text{K}$ could lead comparable sensitivities to state of the art LNAs cooled down to $50\,\text{K}$ [14].

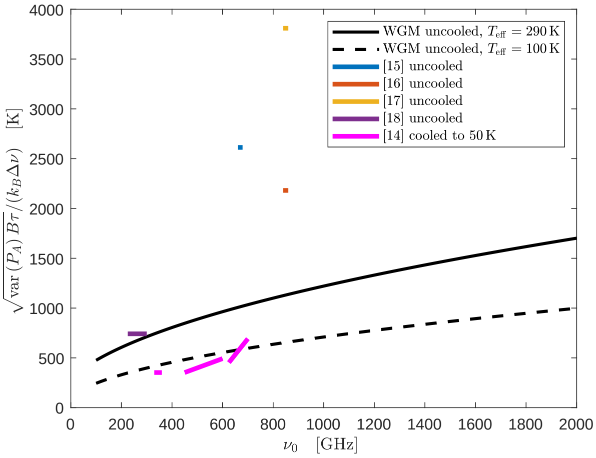

Fig. 1. Radiometer variability of the room temperature upconverter for two different T_e achievable by overcoupling, depending on the intrinsic microwave Q factor of the resonator. A comparison is made with several state of the art receiver schemes reported in the literature.

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