The effect of complex dispersion and impedance in the gain of superconducting traveling-wave kinetic inductance parametric amplifiers

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***Abstract*—Superconducting traveling-wave parametric amplifiers are a promising amplification technology suitable for applications in submillimeter astronomy. Their implementation relies on the use of Floquet transmission lines in order to create strong stopbands to suppress undesired harmonics. For design, amplitude equations are used to predict their gain. However, they do not take into account the complex dispersion and impedance that result from the use of Floquet lines, hindering reliable design. In order to overcome this limitation, we have used the multiple-scales method to include those effects. We demonstrate that complex dispersion and impedance have a stark effect on the gain, even suppressing it completely in certain cases. The equations presented here can, thus, guide to a better design and understanding of the properties of this kind of amplifiers.**

Keywords—Superconductors, parametric amplifiers, gain.

I. Introduction

Achieving larger bandwidths at the RF and IF bands, and improving receiver sensitivity are major challenges for future millimeter and submillimeter heterodyne observations. Consequently, extensive work is being performed in order to improve the performance of SIS junctions and HEMT amplifiers, the key components of current state-of-the-art receivers. However, this approach may soon hit fundamental limitations and improving bandwidths will eventually only be obtained at the expense of sacrificing noise temperature [1]. Recently, a promising superconducting technology that could overcome this problem has emerged [2]. It uses the kinetic inductance (KI) manifest in superconductors to produce parametric amplification in a long transmission line (TL). Devices working with this principle are dubbed Traveling-Wave Kinetic-Inductance Parametric Amplifiers (TKIPAs).

The KI modifies the wave-equation of the current through the TL by adding a nonlinear term which allows mixing of amplitudes when more than one monochromatic signal is injected [3]. Hence, it is possible to amplify the input signal if other tones, called pumps, are simultaneously injected. Nonetheless, more signals, including undesired harmonics, are also generated, deterring the amplification process. Eom et al. solved this problem by implementing a Floquet TL, conformed by a periodically repeating unit cell, creating stopbands that avoid the propagation of the main undesired harmonic of the pump signal. The use of such a line, however, translates into a TL with more intricate properties, namely a complex dispersion and impedance with strong frequency dependencies, particularly close to the stopbands.

In order to design TKIPAs, a nonlinear wave equation must be solved. This is usually done by approximating the process of amplitude gain as a dynamical evolution occurring at a much larger length scale than the wavelength of the involved signals. Within this approximation, not taking into account the complex nature of the Floquet TL, a set of nonlinear amplitude equations can be obtained [2]. An attempt to introduce a complex propagation constant in the process has been reported [4], but some unjustified approximations that are not valid near the stopbands were used.

We have tackled this problem by formally solving the nonlinear wave equation using the *multiple-scales method*, widely used in mathematical nonlinear physics, and especially useful in traveling-wave equations [5]. We demonstrate that the properties of the Floquet TL have a profound effect on the attainable gain, in particular when the pump signal is close to a stopband. Depending on the specific properties of the used Floquet TL and the amplitude and frequency of the pump signal, our equations depart notably from the predictions given by the traditional amplitude equations.

II. Amplitude equations including complex dispersion and impedance

From the telegrapher's equations and the total inductance in a superconductor, $L(I) = L_0(1 + I^2/I_*^2)$, it is found that the dynamics of the electric current I through a TKIPA is described by a nonlinear wave equation,

$$\left(\frac{\partial^2}{\partial z^2} - CL_0\frac{\partial^2}{\partial t^2} - (CR + GL_0)\frac{\partial}{\partial t} - RG\right)I = \frac{L_0}{3I_*^2}\left(G\frac{\partial}{\partial t} + C\frac{\partial^2}{\partial t^2}\right)I^3, \quad (1)$$

where I_* determines the scale of the nonlinear term. C, R, G, and L_0 are the parameters per unit length of the TL used to implement the TKIPA, i.e., capacitance, resistance, conductance, and inductance at zero current, respectively.

Equation (1) can be solved with the multiple-scales method considering that the nonlinear term will produce a modification of the wave amplitudes at a much smaller rate than the wavelength. In order to correctly apply the method

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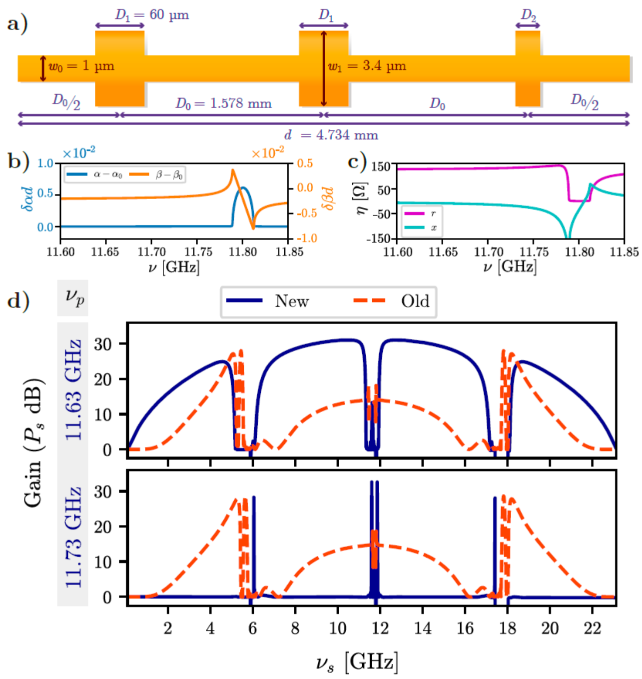

Fig. 1 (a) Unit cell of the used Floquet TL. (b) Dispersion relation near the 2nd stopband, where $\gamma_0 = \alpha_0 + j\beta_0$ is the propagation constant of the central line. (c) Complex impedance near the 2nd stopband. (d) Gain of the signal after traveling $z/d = 150$ unit cells for two values of pump frequency ν_p near the 2nd stopband, using an initial pump amplitude $A_p^0 = 0.2\, I_*$.

at first order, the terms in (1) must be balanced. The balancing depends on the magnitudes of C, R, G, L_0, the propagation constant γ, the frequency ν, and the initial magnitude of current, $|I(z=0)|$. However, the parameters of a Floquet TL depend strongly on ν, especially near the stopbands. This implies that the first order balancing of (1) gives different results at different frequencies. For achieving high amplification, the most relevant case occurs very close to the stopbands. Then, if the multiple-scales method is applied around this frequency and considering three propagating signals, *signal* (s), *idler* (i) and *pump* (p), we obtain

$$\frac{\partial A_{s(i)}}{\partial z} = j g_{s(i)} A_{s(i)} - 2\alpha_{s(i)} A_{s(i)} + \frac{j f_{s(i)}}{8 I_*^2} \times \left[A_{s(i)} \left(\left|A_{s(i)}\right|^2 + 2\left|A_{i(s)}\right|^2 + 2\left|A_p\right|^2 \right) + A_{i(s)}^* A_p^2 e^{j\Delta\beta z} \right], \quad (2.a)$$

$$\frac{\partial A_p}{\partial z} = j g_p A_p - 2\alpha_p A_p + \frac{j f_p}{8 I_*^2} \left[A_p \left(2|A_s|^2 + 2|A_i|^2 + \left|A_p\right|^2 \right) + 2 A_p^* A_s A_i e^{-j\Delta\beta z} \right], \quad (2.b)$$

where $f_m = \frac{1}{2\beta_m}\left(\alpha_m^2 - \beta_m^2 - \frac{|\gamma_m|^2}{|\eta_m|^2}(r_m^2 - x_m^2)\right)$, $g_m = (\alpha_m^2 r_m^2 - \beta_m^2 x_m^2)/(\beta_m(r_m^2 + x_m^2))$, $m = s, i, p$, $\gamma = \alpha + j\beta$ is the propagation constant, $\eta = r + jx$ is the complex characteristic impedance, and $\Delta\beta = 2\beta_p - \beta_s - \beta_i$. Equations (2) differ from the model where α and x are neglected but, importantly, reduce to it.

III. Simulations and Discussion

We designed a CPW Floquet TL (Fig. 1.a) that has stopbands that allow suppressing undesirable harmonics and present a high non-linear dispersion relation $\delta\beta(\nu) \equiv \beta(\nu) - \beta_0(\nu)$ for operation near the stopbands. Fig. 1.b shows this effect around the 2nd stopband. Literature usually neglects x but Fig.1.c shows that it cannot be so since it significantly increases near the stopbands. Indeed, as shown in Fig. 1.d, very different gains of the target signal are obtained when the old (neglecting α and x) and the new model (given by equations 2) are compared. This figure shows two cases of different pump frequency. The case $\nu_p =$ 11.63 GHz shows larger gain with the new model. This occurs because the total phase mismatch of the complex amplitudes stabilizes thanks to the $j g_m A_m$ term in (2). The case $\nu_p = 11.73$ GHz, in contrast, shows almost no gain with the new model. The reason is the $j g_m A_m$ term that dominates over the non-linear one, proportional to f_m.

These results show that amplification larger than predicted by the old model is possible, and that the pump frequency cannot be too close to the stopband in order to achieve amplification.

IV. Conclusions

We have presented a new set of amplitude equations for TKIPAs operating at frequencies near stopbands where, unlike other models, the effect of complex impedance and dispersion have been considered. To highlight the key differences between the models, simulations were performed for a CPW Floquet transmission line, showing that the new model can predict either larger or smaller gain than the traditional one, depending on how close to the stopband is the pump frequency. This happens because one of the new terms added to the amplitude equations is capable of stabilizing the phase mismatch, hence obtaining larger gain, but is also capable of dominate over the non-linear term responsible of amplification, since its magnitude depends on the frequency. Research to experimentally demonstrate these effects is underway.

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