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Astrometry via Moving Reference Sources

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ABSTRACT

Measuring the proper motions of targets via moving reference sources is discussed in context of the lack stationary calibrators in the inner Galaxy. A proof-of-concept of shared astrometry is presented via Monte Carlo simulations of measurements of the proper motions of a generated samples of AGBs. Self-consistent results are obtained, allowing for the prediction of the number of sources needed to achieve error thresholds. Specific sky regions from the BAaDE survey that allow for promising astrometry are identified.

1. INTRODUCTION

Astrometry of the inner galaxy is limited by the lack high-frequency radio quasars to use as calibrators. This paper discusses a proof-of-concept of using moving sources rather than stationary quasars as reference sources to build reference frames. In particular, the known proper motions and errors in proper motions of nearby moving sources can be applied to measure unknown proper motions when the number of quasars is limited or zero. Stellar population were generated from distributions of the positions and motions of Galactic sources to develop a Monte Carlo simulation predicting the expected errors from this method. This paper will refer to moving sources with known proper motions as reference sources and stationary sources as targets.

Critical to this method is the concept of shared astrometry as opposed to absolute or relative astrometry. In absolute astrometry, the position of a source is obtained by measuring its relative position to a reference with a position known accurately. In relative astrometry, the position of the reference sources is not known accurately, and both these procedures can be repeated after time evolution to determine the proper motions of target sources. In contrast, when finding proper motions in shared astrometry, known measurements and measurement errors of proper motions of moving reference sources are shared to measure of the motions of targets.

Section 2 of this paper discusses the method and concept of shared astrometry in further detail, while section 3 discusses simulation parameters for generating the distribution and motions of sources. Section 4 discusses the numerical implementation of the simulation, while section 5 discusses results. Potential applications of this method to determine the unknown proper motions of BaADE (Bulge Asymmetries and Dynamic Evolution) sources are discussed in section 6.

2. CONCEPT

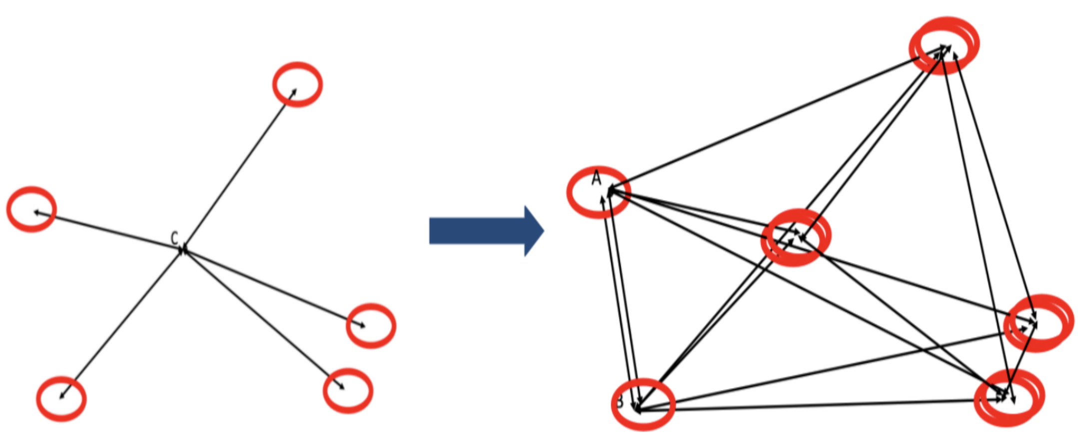

Figure 1. Building a reference frame by measuring relative positions and position errors of all sources respect to each other.

Measuring the proper motions of stars via shared astrometry is done in two steps: first, two reference frames, as viewed from the Earth, are built for two epochs (time-separated, for example, by 5 years), and secondly, these two frames are linked together using known proper motions of a few reference sources to determine the proper motions of many other target sources.

Figure 1 depicts building a single-epoch reference frame with 6 sources. The left-hand image depicts making relative position and position error measurements via a single source. This can be extended to the right-hand image, where every source makes a relative measurement of every other source, which applies to both targets and reference sources. Errors in the relative position (now with respect to the frame) decrease with each measurement. In this fashion, a single-epoch frame can be built with small errors given enough sources. See section 4.1 for a discussion of merging measurement errors.

It should be emphasized that the process of building a single-epoch reference frame is solely dependent on relative astrometry: only relative errors (i.e. observational measurement errors), and not absolute errors (i.e. database errors on the absolute positions of stars), are relevant.

Thus, two reference frames can be built at two different epochs. If given the proper motions of some number of reference sources and two reference frames encapsulating the relative positions between sources, the proper motions of all sources can be determined. In determining these proper motions, one must account for the errors in proper motions of the reference sources – hence the concept of shared astrometry, in which reference source errors are shared amongst target errors.

Using the model described in section 3, a simulation was developed in section 5 generating a sample of moving sources to investigate viability of using shared astrometry to determine proper motions. A certain fraction of the sources were assigned known proper motions and considered to be reference sources, while the remaining proper motions were masked and were treated as targets. In section 6, BaADE data provided target samples with unknown proper motions, while data from a Gaia-BaADE cross-match provided a sample of reference sources.

3. MODEL

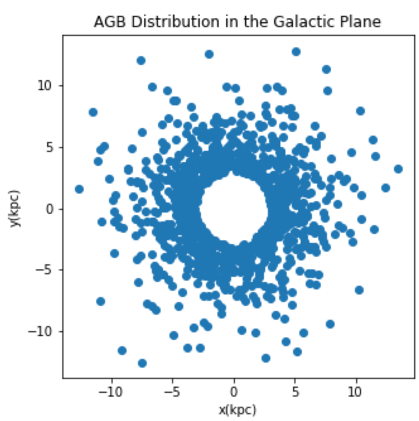

Figure 2. Distributions of AGBs in the Galactic plane generated via a exponentially decaying distribution

Sources were generated by sampling from the positions and proper motions of AGBs. Their positions of AGBs were generated according a exponentially decaying distribution in galactic coordinates (see Quiroga-Nuñez et al. (2017)), and were converted into Ra/Dec coordinates in equatorial coordinates. The distribution of these AGBs in the Galactic plane can be found in Figure 2. Additionally, stellar motions were obtained first by sampling velocities in galactic cylindrical coordinates (V_r, V_ϕ, V_z) each generated from a Gaussian distribution, which is given as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{(x-\rho)}{\sigma})^2} \tag{1}$$

Here ρ refers to the mean (μ here will refer to proper motion) and σ is the standard deviation. These velocities were then converted into proper motions in equatorial coordinates, denoted as μ_{ra} and μ_{dec}. Finally, the errors in

Property	Distribution	Parameters
V_r	Gaussian	$\rho = 0, \sigma = 20km/s$
V_t	Gaussian	$\rho = 240km/s, \sigma = 20km/s$
V_z	Gaussian	$\rho = 0, \sigma = 5km/s$
$\sigma_{\mu_{ra}}$	Skewed Gaussian	$\rho = 0.96, \sigma = 8.41, \alpha = 24.9$ (%)
$\sigma_{\mu_{dec}}$	Skewed Gaussian	$\rho = 1.8, \sigma = 5.7, \alpha = 5.98$ (%)

Table 1. Parameters for generating proper motions of AGBs. Motions in the Galactic plane and percent errors in the equatorial plane are shown. Note that any assisting generated quasars are considered to have zero effective proper motions and proper motion errors.

these proper motions, denoted by $\sigma_{\mu_{ra}}$ and $\sigma_{\mu_{dec}}$ were generated by sampling percent errors from a skewed Gaussian distribution, which, when centered around zero mean and unit standard deviation, is as follows:

$$f(x) = 2\phi(x)\Phi(\alpha x) \tag{2}$$

where $\phi(x)$ is the probability density function, $\Phi(x)$ is the cumulative distribution function, and α is the skewness parameter. Parameters for these skewed Gaussian error distributions were obtained from fitting Gaia data. See Table 1 for a summary for the generation of proper motions and proper motion errors. Quasars, if any (≤ 1) are to be used as simulated reference sources, can be generated with zero proper motion and proper motion error.

In section 5, BaADE target sources are assigned random proper motions following the same distribution of velocities in Table 1, with the additional assumption that they are located more than 2 kpc from the center of the galaxy.

4. PROCEDURE

A summary of the implementation of the simulations is as follows: in the first epoch, sources with positions, proper motions (known for references, unknown for targets), and proper motion errors is generated. Next, a reference frame is built via relative measurement between all sources. To transition from the first epoch to the second epoch, the positions of the sources are time-evolved given the generated proper motions. Then, a reference frame is built in the second epoch. Finally, by comparing the reference frames for different epochs and applying reference source proper motions and proper motion errors, target proper motions and their errors are determined. Section 3 contains details for generating sources, while section 4.1 contains the numerical details of error handling and section 4.2 details building and linking frames.

4.1. *Error Handling*

This subsection will discuss the generation error propagation from variable uncertainty and combination from multiple measurements.

Given some function of many variables $f(x_1, x_2, ...)$ and errors associated with those variables δx_i, the error of the function can be found via the following formula:

$$(\delta f)^2 = \Sigma_i (\frac{\partial f}{\partial x_i} \delta x_i)^2 \tag{3}$$

For example, the error in velocity from $v = \frac{x(t_1) - x(t_0)}{\Delta t}$ with zero-error Δt is $\delta v = \frac{\sqrt{(\delta x(t_1))^2 + (\delta x(t_2)^2}}{\Delta t}$. When many measurements $(x_i, \delta x_i)$ are made of one object, an average can be obtained as follows weighting according to errors:

$$x = \frac{\Sigma_i \frac{x_i}{\delta x_i^2}}{\Sigma_i \frac{1}{\delta x_i^2}} \tag{4}$$

Using the error propagation formula, the error on this average measurement is

$$\delta x = \sqrt{\frac{1}{\Sigma_i \delta x_i^{-2}}} \tag{5}$$

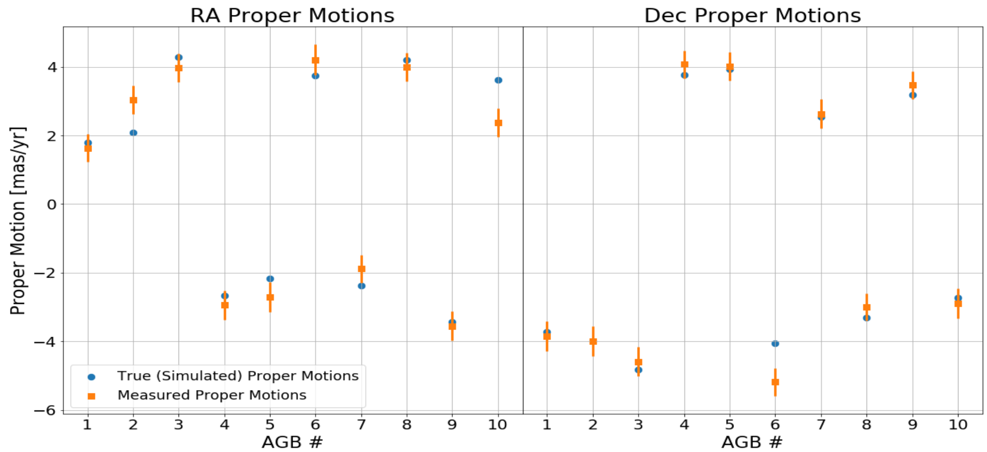

Figure 3. Results of a single simulation using 1 reference sources to measure the proper motions of 10 targets. Errors are small relative to proper motions and measurements lie within acceptable deviation from true values.

4.2. *Building and Linking Epoch Frames*

In each epoch a separate reference frame is created via relative measurement. As an example, consider two sources A and B, which are generated with absolute positions (ra, dec) via the model in section 3. The act of measurement is simulated assuming Gaussian observation errors with mean $\rho = 5mas$ and standard deviation $\sigma = 1mas$ for both right ascension and declination. The measurement of relative difference in declination from A to B is simulated simply by computing $dec_B - dec_A$, but the relative difference in right ascension contains an additional cosine term, and is of the form $(ra_B - ra_A)\cos(dec_B)$ (and the same process is done measuring relative differences from B to A). Thus, given N total sources, there are $N(N-1)$ total relative measurements per epoch.

The next step in building these frames is combining multiple measurements of a single source together. To do so, shared astrometric positions are obtained. For example, continuing with the two sources A and B, the shared astrometric declination of B from A is $dec_A + \Delta dec_{BA}$). All measurements of these declinations are combined via equations 4 and 5 in section 4.1. It should be emphasized that these shared positions are not absolute positions, and hence no absolute errors should be included.

However, shared positions will be used to compute proper motions, where $\mu_{dec} = \frac{dec(t_2) - dec(t_1)}{dt}$, with dt typically around 5 years. μ_{ra} is computed in a similar fashion accounting once again for an additional cosine term, and the respective proper motion errors are computed via equation 5 in section 4.1. Thus, since these shared positions are used to compute proper motions, the errors in the shared positions in epoch 2 must combine both measurement errors and known proper motion errors via Equation 3.

5. SIMULATION RESULTS

Simulations received the number of reference sources, the number of targets, and the number of quasars (≤ 1) as input parameters. Sources were selected from a simulated distribution of sources using the procedure in Section 3 across the Milky Way given a field of view of 5 degrees.

Result of a single simulation are shown in Figure 3, where 1 reference source was used to build a frame and determine proper motions of 10 targets. Simulated measurements are obtained with errors in acceptable limits ($\leq 1mas/yr$).

This procedure can be extended as a Monte Carlo simulation to predict expected proper motion errors given varying numbers reference sources. Results of this procedure are shown in Figure 4. For each number of reference sources, 100 independent sets of sources were generated with 10 targets. Astrometry was performed with each of these independent sets using the procedure outlined in section 4, and resulting errors were averaged among these 100 realizations. Given the plots of Figure 4, the number of reference sources needed to perform astrometry given a certain error tolerance can be predicted.

After this Monte Carlo simulation is generated, one can determine how many standard deviations measurements deviate from their true values. Let the Z-score be defined as follows:

$$Z = \frac{x_1 - x_0}{\sigma} \tag{6}$$

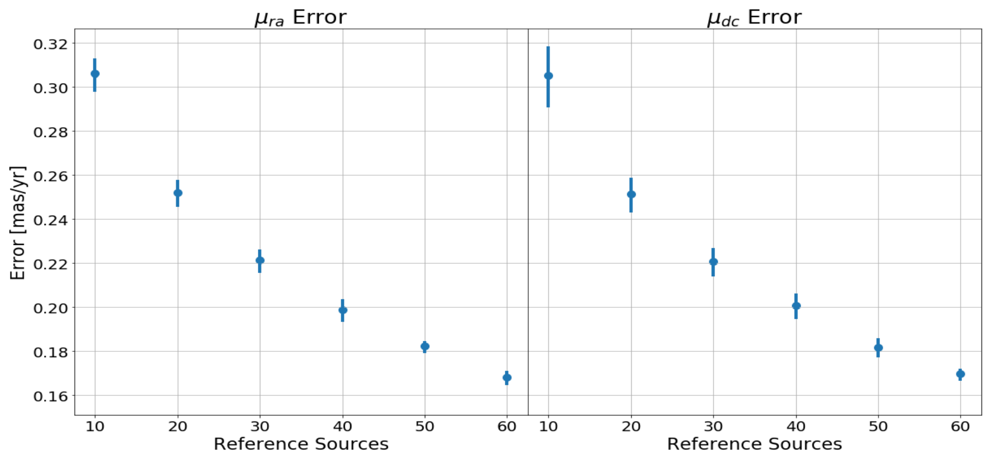

Figure 4. Average errors obtained in a Monte Carlo investigating expected proper motion errors as a function of the number of reference sources. Spread of errors ("errors of errors") are included as error bars.

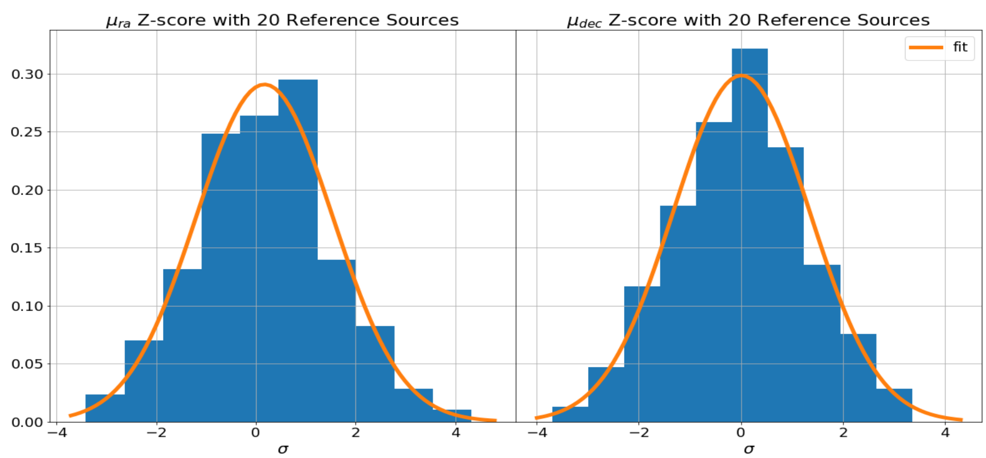

Figure 5. Normalized distribution of the number of standard deviations measurements are from their true (simulated) values and a fit to a Gaussian distribution.

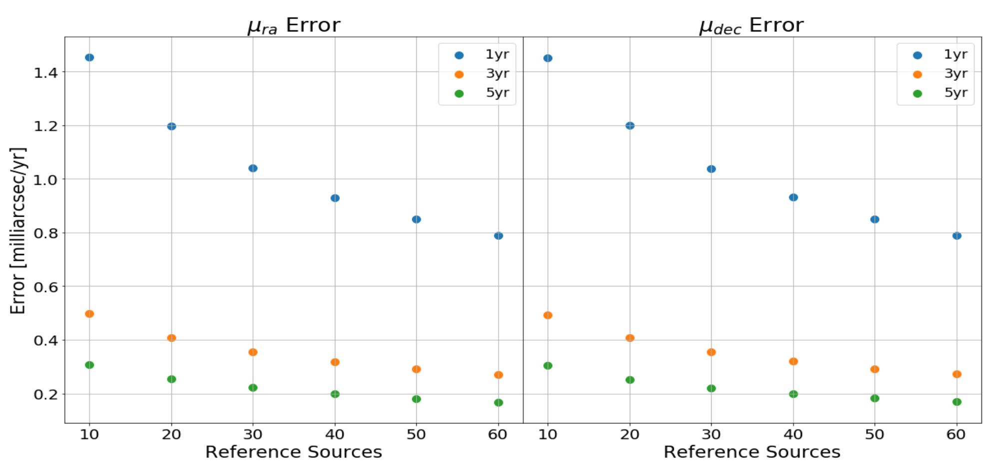

Figure 6. Results from the Monte Carlo simulation as a function of time. Self-consistent decreasing errors with increasing number of sources and time.

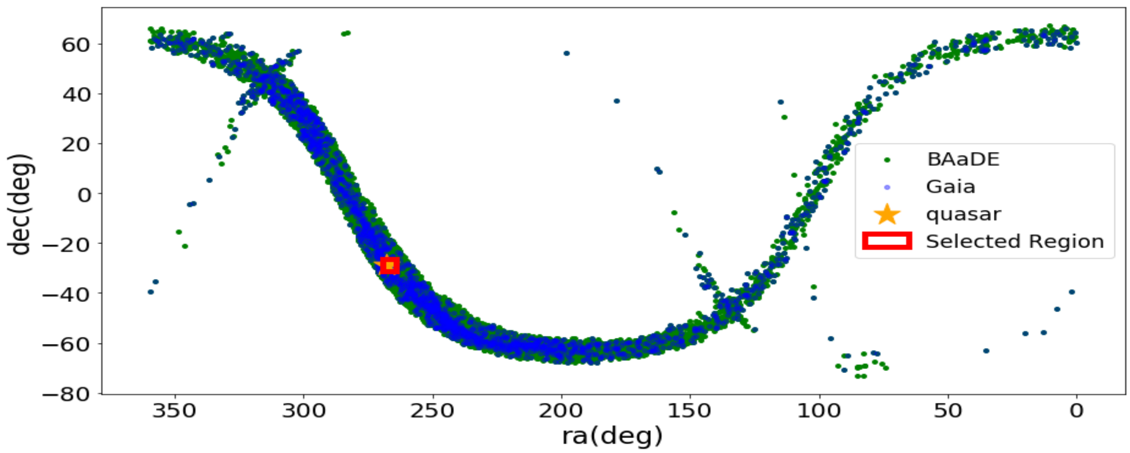

Figure 7. Distribution of sources from the Gaia and BAaDE database in the equatorial plane. Selection of a region containing a quasar and a high density of reference sources.

where x_1 is the observed value, x_0 is the true value, and σ is the measurement error. A z-score distribution is shown in Figure 5 and is appropriately Gaussian. A fit of these distributions with varying number of reference sources is shown in Table 2, with approximately zero mean and consistent standard deviation across the Monte Carlo. Finally, the time separation between the two epochs can be changed; the result of this is shown in Figure 6, and the error appropriately decreases with increasing time. These consist results across the Monte Carlo demonstrate confidence in the numerical implementation of error propagation.

Known	μ_{ra}	σ_{ra}	μ_{dec}	σ_{dec}
10	0.003	1.36	0.16	1.40
20	-0.03	1.32	0.04	1.44
30	-0.06	1.33	0.05	1.40
40	-0.15	1.35	0.11	1.39
50	-0.05	1.41	0.04	1.37

Table 2. Fit of z-score distributions of proper motion measurements relative to their true values to Gaussian distributions

6. APPLICATIONS

This section examines potential sky regions of interest to apply astrometry via moving sources to determine target proper motions. The BAaDE survey has obtained the positions 20000 sources without obtaining their proper motions, while a BAaDE-Gaia (afterwards referred to as just Gaia) cross-match has obtained a sample of 2000 sources with known proper motions and proper motion errors (Quiroga-Nuñez et al. (2020)). Sources from BAaDE and the Gaia cross-match can be treated as targets and reference sources, respectively.

Table 3 investigates several potential regions of interest within a 5 degree field of view in these datasets. Fast moving sources have allow for less time separation between two epochs, and quasars with zero proper motion can help reduce measurement errors. Regions of high density enable the construction of frames with more sources to further reduce measurement errors and allow for easier phase referencing to account for atmospheric effects.

A region containing quasar J1745-2900 was selected and expected proper measurement errors for BAaDE sources within the region were obtained. Positions, proper motions, and proper motion errors for reference sources were obtained from Gaia. Target positions were obtained from the BAaDE database, but their proper motions and proper motion errors had to be generated via the sampling method of Section 3. The results for this simulation are shown in Figures 8 and 9. As simulated proper motion measurements lie within 3 standard deviations of true simulated proper motion values, this study demonstrates that shared astrometry can be applied to make observations with radio interferometers with sufficiently long baselines for the angular resolution required.

7. CONCLUSION

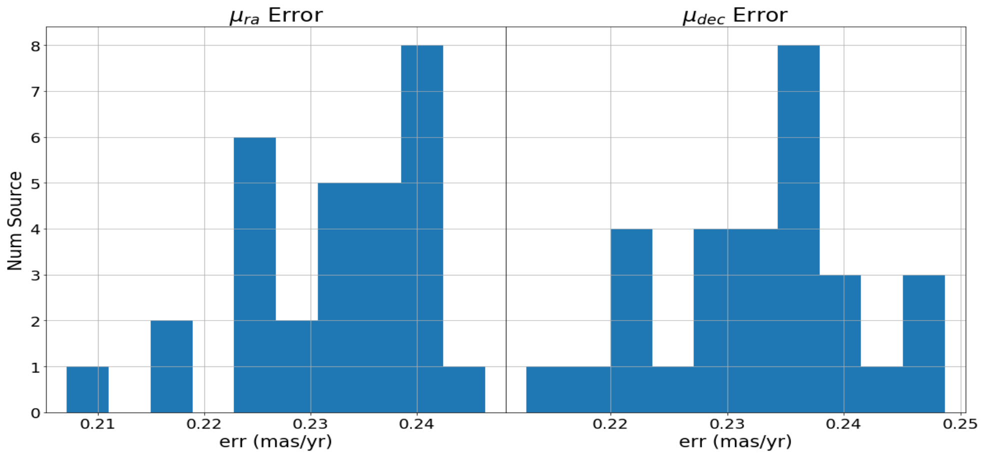

Figure 8. Distribution of errors for simulated proper motions of BAaDE sources via calibration with Gaia sources.

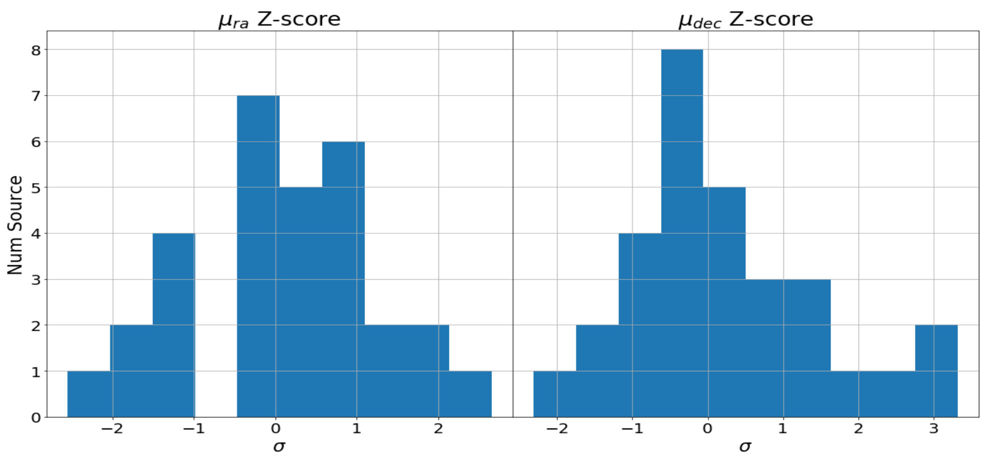

Figure 9. Deviations from mean of simulated proper motion measurements of BAaDE sources.

Region	Ra [deg]	Dec [deg]	Gaia Sources	BAaDE Sources
High Velocity	214.719	-61.53	25	165
Contains Quasar	268.08	-28.99	31	1045
High Gaia/Baade Ratio	243.56	-53.48	58	217
High Density	266.66	-28.95	48	2599

Table 3. 4 sky regions of interest with selection criteria for obtaining the best astrometry.

This paper illustrates a proof of concept of determining proper motions via moving reference sources through a shared astrometry method. Distributions of the Galactic positions and motions of sources were generated and sampled from to simulate the evolution of a sky field of view. Measurement by building reference frames in two epochs and linking together to determine the proper motions of targets via reference sources was simulated. This method is best classified as shared astrometry, as opposed to relative or absolute astrometry, as the proper motions and errors of reference sources were applied to determine those of targets.

The number of reference sources necessary to obtain errors within a certain tolerance was investigated via Monte Carlo simulations. The results were self consistent, with proper motion measurements lying in a Gaussian distribution about their true values and errors falling under acceptable ranges with an easily obtainable number of reference sources. This proof of concept can be extended to investigate sky regions of interest. By obtaining reference sources from Gaia data and simulating the proper motions of BAaDE sources, it was shown that consistent and accurate proper motions can be obtained.

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