Neutron Star Radiation-Driven Winds

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ABSTRACT

A significant fraction of observed type I X-ray bursts exhibit photospheric radius expansion (PRE). This phenomenon has become an important means of measuring neutron star radii and masses, which often lead to constraints on the dense matter equation of state. To interpret observations accurately, the details of the expansion and possible subsequent outflow need to be understood. We review the general relativistic radiation hydrodynamics equations to model PREs as winds, starting from their general covariant form and outlining the derivation of the final mixed-frame equations usually solved. We then build models for spherically symmetric, optically thick, steady-state winds of neutron stars of mass $M = 1.4 M_\odot$ and radius $R = 12$ km with various mass-loss rates. We focus on describing the relative importance of the radiation and gas pressures throughout the wind and its implications. We find that ejection of matter happens when the opacity increases enough to lower the local Eddington luminosity below the local luminosity. In our models, this first happens between $5 - 9 R$; the luminosity exceeds the Eddington luminosity by at most $\sim 0.014 - 0.25\%$ and only $\sim 0.001 - 0.02\%$ of the accreted matter, up to the photosphere, is ejected into infinity.

1. INTRODUCTION

Type I X-ray bursts are high energy transients that typically last tens of seconds and are product of thermonuclear runaway on the surface of accreting neutron stars. The emerging radiation flux can expand the outer atmospheric layers of the star in what is called a photospheric radius expansion (PRE). Around 20% of type I X-ray bursts show PRE as determined from the shift of their emission peak (Guichandut et al. (2021)). If powerful enough, the photospheric expansion can ultimately eject matter into a wind. PREs have been extensively used (e.g., Özel (2006); Güver et al. (2010); Özel & Freire (2016)) to make neutron star radius and mass measurements and subsequently place constraints on the dense matter equation of state (Steiner et al. (2010)). In order to properly interpret observations, the dynamics of the expansion needs to be carefully understood.

The first attempts to model these radiation-driven winds used Newtonian gravity and assumed time independence (e.g., Quinn & Paczynski (1985); Joss & Melia (1987)). Soon, Paczynski & Prosynski (1986) included general relativity (GR) in steady-state, optically thick wind models by using the equations of stationary spherical accretion into blackholes by Thorne et al. (1981) and Flammang (1982) derived from the PSTF radiation moments formalism developed by Thorne (1981). Their (Paczynski & Proszyński (1986)) models showed general relativistic effects were decidedly important in determining the wind structure, especially at low mass-loss rates.

More recently, Yu & Weinberg (2018) performed the first time-dependent simulations using MESA and also explored the evolution of the wind composition, showing that ashes of nuclear burning could represent a significant portion of the final composition. Meanwhile, Herrera et al. (2020) focused on finding correlations between photospheric quantities and the mass and energy-loss rate parameter space. Both of these works modeled optically thick, Newtonian winds. On the other hand, Guichandut et al. (2021) modeled winds within the context of GR and static expanded atmospheres, previously explored by Paczynski & Anderson (1986), and extended their models to optically thin regions by using flux-limited diffusion to describe the radiative transfer.

This work is the first phase of a bigger effort to drop the assumptions commonly made. As a first step, we build general relativistic models of a steady-state, spherically symmetric, optically thick wind. Our ultimate goal is to use this work to test these usual assumptions with future general relativistic models that will explore the temporal and angular dependencies of the wind. We begin by outlining the derivation of the radiation hydrodynamics equations that are usually solved to build GR wind models in Section (2). In Section (3), we present the assumptions, final equations, and boundary conditions that shape our models and describe our method.
The covariant, flux-conservative form of the radiative transfer equation can be derived from the Boltzmann equation for photons

\[
(n^\alpha I_\nu/\nu)_\alpha = \frac{\partial}{\partial \nu} \left( n_\nu I_\nu/\nu \right) = \frac{1}{\sin \zeta} \frac{\partial}{\partial \zeta} \left( n_\zeta I_\nu/\nu \right)
\]

\[+ \frac{\partial}{\partial \psi} \left( n_\psi I_\nu/\nu \right) = j_\nu - \alpha_\nu I_\nu \tag{5}\]

where, for an arbitrary spacetime, \( n_\nu, n_\zeta, \) and \( n_\psi \) are defined by unit direction vectors in the tetrad basis (Equation (4)) and the connection coefficients (see Davis & Gammie (2020)) for a complete derivation. In the source term we find the emissivity \( j_\nu \) and the extinction coefficient \( \alpha_\nu \), which are properties of the interacting matter. Equation (5) can be rewritten as

\[
(n^\alpha n_\beta I_\nu)_\alpha = \frac{\partial}{\partial \nu} (n_\nu n_\beta I_\nu) - \frac{1}{\sin \zeta} \frac{\partial}{\partial \zeta} (n_\zeta n_\beta I_\nu)
\]

\[+ \frac{\partial}{\partial \psi} (n_\psi n_\beta I_\nu) = n_\beta (j_\nu - \alpha I_\nu) \tag{6}\]

By integrating over frequency and solid angle, we obtain the expression for conservation of stress-energy

\[R_{\alpha\beta} = -G^\alpha \tag{7}\]

where \( G^\alpha \) is the radiation four-force density

\[G^\alpha = \frac{1}{c} \int \int n^\alpha (\alpha_\nu I_\nu - j_\nu) d\nu d\Omega \tag{8}\]

(Park (2006)). If only the matter-radiation interaction is present, the total energy and momentum are conserved

\[(T^{\alpha\beta} + R^{\alpha\beta})_{\alpha\beta} = 0 \tag{9}\]

From Equation (7), this implies

\[T_{\alpha\beta} = G^\alpha \tag{10}\]

Equations (7) and (10) illustrate how the radiation four-force density couples radiation to matter: the energy and momentum lost by the radiation field is gained by the gas, and the converse is also true.

2.1.1. Schwarzschild spacetime

The equations in §2.1 are applicable to an arbitrary spacetime. We model a spherically symmetric outflow in a Schwarzschild spacetime where the metric is characterized by the neutron star mass \( M \) and can be expressed
We set $c = 1$ in the time coordinate.
\[
\left(1 - \frac{2GM}{c^2r}\right)^{-1} \frac{\partial \dot{U}_{rad}}{\partial t} + \left(1 - \frac{2GM}{c^2r}\right)^{-1} r^{-2} \times \frac{\partial}{\partial r} \left(F^r r^2 \left(1 - \frac{2GM}{c^2r}\right)\right) = -c\xi \left(1 - \frac{2GM}{c^2r}\right)^{-1} \times \left(G^i + v\bar{\alpha}F^i/c\right)
\]

where \(\dot{U}_{rad}\) is the radiation energy density in the tetrad frame.

\section{MODEL DESCRIPTION}

We now model an optically thick, spherically symmetric, steady-state wind. With these assumptions, the radial Equations (16)-(19) can be manipulated to obtain an expression for the mass-loss rate \(\dot{M}\), an equation of motion, an expression for the energy-loss rate \(\dot{E}\), and the radiative transport equation in the diffusion approximation, all in cgs units.

\[
\dot{M} = 4\pi r^2 \rho v \xi
\]

(20)

\[
\frac{dP}{dr} = -(\rho c^2 + U + P) \frac{d\ln \xi}{dr}
\]

(21)

\[
\dot{E} = L_{\infty} + \xi H \dot{M}
\]

(22)

\[
\frac{d\ln T}{dr} = - \frac{3\kappa \rho L_{\infty}}{16\pi acr^2 T^4} \left(1 + \frac{v^2}{c^2}\right)^{-1} \xi^{-3} - \frac{d\ln \xi}{dr}
\]

(23)

We drop the notation used in §2.1.1 to distinguish between reference frames and clarify here that the mass and energy-loss rates are measured in the coordinate frame while the temperature \(T\), density \(\rho\), opacity \(\kappa\), total pressure \(P = P_g + P_{rad}\), total energy density \(U = U_g + U_{rad}\), and total enthalpy per unit mass \(H\) are measured in the comoving frame. For an ideal gas in local thermodynamic equilibrium (LTE) with an isotropic radiation field, the last three quantities are defined as follows:

\[
P = \frac{k_B T \rho}{\mu m_a} + \frac{1}{3} a T^4
\]

(24)

\[
U = \frac{3}{2} \frac{k_B T \rho}{\mu m_a} + a T^4
\]

(25)

\[
H = c^2 + U + \frac{P}{\rho}
\]

(26)

where \(m_a\) is the atomic mass unit. We model winds composed of fully ionized helium, setting the mean molecular weight to \(\mu = 4/3\). Yu & Weinberg (2018) investigated the evolution of the composition in their Newtonian simulations showing that, depending on the ignition depth of the burst, the wind can be dominated by light elements like \(^4\)He or by ashes of nuclear burning like \(^{48}\)Cr, \(^{52}\)Fe and \(^{56}\)Ni. It is not expected that composition will significantly alter the structure of optically thick winds; however, when extending the models to optically thin regions, absorption by heavy elements may become important.

At the high temperatures maintained by the burst, electron scattering dominates the matter-radiation interaction. We adopt the scattering formula by Paczynski (1983) which takes into account Klein-Nishina corrections, while ignoring their degeneracy term as the winds are not degenerate at any point (see Paczynski & Proszynski (1986)).

\[
\kappa = 0.2 \left[1 + \left(\frac{T}{4.5 \times 10^{8} \text{K}}\right)^{0.86}\right]^{-1}
\]

(27)

The comoving luminosity \(L\) is related to the luminosity measured by an observer at infinity \(L_{\infty}\) i.e., the coordinate frame luminosity, by a frame transformation

\[
L = L_{\infty} \left(1 + \frac{\dot{v}^2}{c^2}\right)^{-1} \xi^{-2}
\]

(28)

Moreover, it is useful to define a local Eddington luminosity in the comoving frame (Paczynski (1983))

\[
L_{Edd} = \frac{4\pi cGM}{\kappa} \left(1 - \frac{2GM}{c^2r}\right)^{-1/2}
\]

(29)

Equations (13), (20), (21), (23), and (24) can be manipulated to give a system of coupled ordinary differential equations for the velocity and temperature,

\[
\frac{d\ln v}{d\ln r} = \frac{GM}{r} \left(1 - \frac{2GM}{c^2r}\right)^{-1} \left(1 + \frac{c_s^2}{2v^2}\right) - 2c_s^2 - C \left(\frac{c_s^2 - v^2}{a}\right)
\]

(30)

\[
\frac{d\ln T}{d\ln r} = - \frac{3\kappa \rho r F}{4aT^4\xi} - \frac{GM}{c^2r} \left(1 - \frac{2GM}{c^2r}\right)^{-1} - \frac{\gamma^2 v^2}{c^2} \frac{d\ln v}{d\ln r}
\]

(31)

where \(F\) is the comoving radiative flux, and a third equation describing the density of the gas can be derived by means of Equations (13) and (30)

\[
\frac{d\ln \rho}{d\ln r} = \left(2\frac{v^2}{r\xi^2} + C\right) \left(c_s^2 - v^2\right) A
\]

(32)
where

\[
A = 1 + \frac{3}{2} \frac{c_s^2}{c^2}
\]

and

\[
C = \frac{\kappa L}{16 \pi \nu c r} \left( \frac{4 - 3 \beta}{1 - 3 \beta} \right) \left( 1 + \frac{v^2}{c^2} \right)^{-1} \xi^{-3}
\]

The isothermal sound speed, and \( \beta = P_g/P \) is the ratio of the gas pressure to the total pressure. Equations (20)-(34) are the same as those in Paczynski & Proszynski (1986), except for Equation (29) which they call “critical luminosity”.

An important feature of the velocity equation (Equation (30)) is the discontinuity at \( v = c_s/\sqrt{A} \), that is, when the flow velocity is not exactly but slightly below the sound speed due to general relativistic effects \((\sim 0.9999995 c_s \text{ Paczynski & Proszynski (1986))})\). We refer to the location where this discontinuity happens as the critical radius or critical point \( r_{cr} \). In order to integrate through the critical point we adopt the change of variable by Guichandut et al. (2021):

\[
\Phi = A^{1/2} \frac{v}{c_s} + A^{-1/2} \frac{\xi}{v}
\]

its gradient now substitutes Equation (30)

\[
\frac{d\Phi}{dr} = - \left[ \frac{GM}{r} \left( 1 - \frac{2GM}{c_s^2 r} \right)^{-1} \left( 1 + \frac{c_s^2}{2c^2} \right) - 2c_s^2 - C \right]
\times \left( \frac{v \gamma c_s A^{1/2}}{c_s^2} \right)^{-1} + \left( A \frac{v^2}{c_s^2} - 1 \right) \left( 3c_s^2 - 2Ac^2 \right) \frac{d\ln T}{d\ln r}
\times \left( \frac{4v}{c_s A^{3/2} c_s^2 r} \right)^{-1}
\]

where \( \gamma = (1 - v^2/c^2)^{-1/2} \) is the Lorentz factor, and the discontinuity at the critical point has been removed. One can then compute the velocity from \( \Phi \)

\[
v = \frac{c_s}{2\sqrt{A}} \left( \Phi \pm \sqrt{\Phi^2 - 4} \right)
\]

where the negative branch corresponds to \( v(r < r_{cr}) \), and the positive branch to \( v(r > r_{cr}) \). We also ignore the last term in Equation (31) as the winds are non-relativistic \( (v^2 \ll c^2) \).

A physical solution to the wind problem must pass through the critical point and satisfy boundary conditions. To ensure the former, we begin integrating \( dT/dr \) and \( d\Phi/dr \) from the critical point inward and from the critical point outward. The boundary conditions determine the correct values of the parameters, which we choose to be the energy-loss rate \( \dot{E} \) and the critical radius \( r_{cr} \). To construct a model, we fix the mass-loss rate \( \dot{M} \) and probe a parameter space; for each pair \( (\dot{E}, r_{cr}) \) the following algorithm is attempted.

At the critical point, \( \Phi = \Phi_{cr} = 2 \) and the temperature \( T_{cr} \) is computed by finding the root of the numerator in Equation (30) as at this point, it must vanish along with the denominator. The luminosity at infinity can then be computed from Equation (22), and one can subsequently solve the system of Equations (31) and (36). For the inward integration, we follow the approach by Guichandut et al. (2021) and solve Equations (31) and (36) from the critical point \( r_{cr} \) to 0.90 \( r_{cr} \) and switch to integrating radius and temperature changes with respect to density changes. The corresponding system of equations \( dr/d\rho \) and \( dT/d\rho \) can be obtained from Equations (31) and (32). Near the surface of the star, the wind structure is quasi-static, meaning hydrostatic equilibrium is a good approximation. In hydrostatic equilibrium we have the pressure condition \( P = g y \) where \( g \) is the gravitational acceleration at the surface of the star and \( y \) is the column depth. An adequate choice for the surface gravity can be derived from the equation of motion (Equation (21)) by setting the non-relativistic limits \( v \ll c \) and \( (U + P)/\rho \ll c^2 \) in the enthalpy term; this yields \( g = \frac{GM}{R^2} \left( 1 - \frac{2GM}{c_s^2 R^2} \right)^{-1} \). Note that Guichandut et al. (2021)) use the same pressure condition but with a weaker surface gravity. We require this pressure condition to be satisfied at the surface of the neutron star with a column depth \( y = 10^8 \text{ g cm}^{-2} \) which is close to typical ignition depths for type I X-ray bursts (Galloway & Keek (2021)). This defines our inner boundary condition:

\[
y = P/g = 10^8 \text{ g cm}^{-2} \text{ at } r = R
\]

If the \((\dot{E}, r_{cr})\) pair yields a successful inward integration, i.e., one that satisfies Equation (38), we proceed to integrate outward. Similarly, we solve Equations (31) and (36) from the critical point out until the photosphere is reached. We define the photosphere as the location \( r_{ph} \) where the luminosity is given by the Stefan-Boltzmann law, which is valid for optically thick media. To ensure we stay within the optically thick region of the wind, we impose a value of the optical depth parameter \( \tau^* = k\rho r \) at the photosphere. Quinn & Paczynski (1985) found self-consistent Newtonian models for \( 3 < \tau^* < 5 \). Given that this condition is satisfied several tens to hundreds of kilometers away from the neutron star, this value should be similar in the general relativistic case. Indeed, Guichandut et al. (2021) found \( 3.7 \lesssim \tau^* \lesssim 3.9 \) at \( r_{ph} \). Based on their findings, we require \( \tau^*(r_{ph}) = 3.8 \).
This sets our outer boundary condition:

\[ L = 4 \pi \sigma r^2 T^4 \] at \( \tau^* = k \rho r = 3.8 \) \hspace{1cm} (39)

It should be noted that the actual value of the optical depth at the photosphere requires a more elaborate treatment of the radiative transfer.

If the outward integration satisfies Equation (39), the \((\dot{E}, r_{cr})\) pair is optimized to reduce residuals on the boundary conditions. In the case that more than one pair of parameters completes this algorithm successfully, the pair that best satisfies the boundary conditions after optimization is chosen as the final model for that mass-loss rate. The outcome of solving this system of ordinary differential equations is highly dependent on the chosen parameters; therefore, to find physical solutions, the parameter space needs to be exhaustively sampled. Recently, Herrera et al. (2020) investigated the Newtonian parameter space and found correlations between model-defining quantities. Though not included here, we found that for a common mass-loss rate and boundary conditions, a satisfactory Newtonian model has a lower energy-loss rate and reaches the critical point closer to the neutron star surface than a general relativistic model.

4. RESULTS

4.1. Wind profiles

We constructed wind models for neutron stars of mass \( M = 1.4 \text{M}_\odot \) and radius \( R = 12 \text{ km} \) per recent constraints (e.g., Drischler et al. (2021); Raaijmakers et al. (2021)) considering five mass-loss rates \( \dot{M} = 10^{17.25}, 10^{17.5}, 10^{17.75}, 10^{18}, 10^{18.25} \text{ g s}^{-1} \). In Figure (1) we plot their velocity, temperature, density and luminosity profiles, where × s mark the critical points and filled circles (●) mark the photospheres. Close to the surface of the star, the wind structure is quasi-static with velocities of a few cm s\(^{-1}\); the temperature is around a billion kelvin, which lowers the electron scattering opacity significantly, allowing for a large outflow of radiation; and densities are of several tens of thousands g cm\(^{-3}\). In panel c) the luminosity is observed to decrease slightly with radius because of gravitational redshift and work done on the gas. In general, winds with higher mass-loss rates are more extended, ending with lower photospheric temperatures and densities than models with lower mass-loss rates. Winds with higher mass-loss rates also reach greater velocities at the photosphere and correspondingly have lower radiation output. All models reach the critical point within \( 3.7 - 6.7 R \), where the winds are highly optically thick \((17 \lesssim \tau^* \lesssim 98)\), and have photospheric radii between 10 to 20 \( R \). At the photosphere, the wind velocities reached are non-relativistic with values between \( \sim 0.002 - 0.006 \text{c} \), and the temperatures and densities have decreased to \( \sim 7.5 - 4 \text{ MK} \) and \( \sim 15 - 5 \times 10^{-7} \text{ g cm}^{-3} \), respectively. We stopped exploring lower mass-loss rates because, as the winds become optically thin at the critical point, finding acceptable solutions becomes increasingly demanding of precise parameter values. Our upper limit in mass-loss rate was set by nuclear energy release considerations (Guichandut et al. (2021)).

4.2. Radiation-driven winds
Figure 2: Relative importance of the gas and radiation pressures at the base (top panel) and throughout the wind (bottom panel).

Figure (2) shows the relative importance of the gas pressure to the radiation pressure. The top panel shows the contribution of the gas pressure to the total pressure at the base of the wind as a function of mass-loss rate. At the base, the gas and radiation pressure contribute almost evenly in the log $\dot{M} = 17.75$ model, while the log $\dot{M} = 17.25$, 17.5 models are dominated by gas pressure and the log $\dot{M} = 18$, 18.25 models, by radiation pressure. This trend is the result of higher base luminosities in winds with higher mass-loss rates (see panel d) in Figure (1)). The bottom panel shows that even for the models in which gas pressure dominates at the base, radiation pressure quickly exceeds gas pressure and remains 3–4 orders of magnitude higher throughout the extent of the wind, showing these winds are accelerated almost entirely by radiation. As winds with high mass-loss rates maintain higher densities, the gas pressure maintains a relatively higher importance than in the low mass-loss rate models.

The local Eddington luminosity defined in Equation (29) requires the assumption of a pseudo-Newtonian potential. Although not strictly correct in GR, our use of it is at far enough distances from the point mass $M$ (> 22 gravitational radii) that it can provide useful insight into the ejection of matter by the radiation force. In Figure (3), we plot the comoving luminosity in terms of the Eddington luminosity as a function of radius. Close to the surface of the neutron star, the high temperatures lower the opacity, which results in the luminosity being strongly sub-Eddington. The luminosity approaches the Eddington luminosity as the latter decreases with increasing opacity further away from the surface. Between $\sim 15–20$ kilometers after the critical point, the luminosity reaches and exceeds the Eddington luminosity by less than 0.25% for most models. It is until this point is reached that the radiation force can eject matter into infinity. The model with the lowest mass-loss rate log $\dot{M} = 17.25$ is never super-Eddington; however, it comes close enough ($0.999986 L_{Edd}$) that the force due to the gas pressure, although small, can contribute enough so that some matter is ejected into a wind.

We can compute the enclosed mass between two points as measured in the coordinate frame by

$$m = 4\pi \int_{r_0}^{r} r^2 \rho \left(1 - \frac{2GM}{c^2 r} \right)^{-1/2} dr$$  \hspace{1cm} (40)

We define $m_{in}$ as the mass enclosed between the wind base and the critical point and $m_{out}$, as the mass en-
The enclosed mass between the wind base and the critical point $m_{\text{in}}$, between the critical point and the photosphere $m_{\text{out}}$, and between the point where the luminosity becomes super-Eddington $r_{\text{L,edd}}$ and the photosphere as a function of mass-loss rate.

Given that the winds are radiation-driven (as shown in Figure (2)) we can approximate the ejected mass $m_{\text{ej}}$ to that ejected by the radiation force, i.e., the mass enclosed from the point $r_{\text{L,edd}}$ where $L = L_{\text{edd}}$ out to the photosphere.

$$m_{\text{ej}} \approx 4\pi \int_{r_{\text{L,edd}}}^{r_{\text{ph}}} r^2 \rho \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} dr \quad (41)$$

Figure (4) shows $m_{\text{in}}$, $m_{\text{out}}$, and $m_{\text{ej}}$ as a function of mass-loss rate. $m_{\text{in}}$ is fairly constant across mass-loss rates, while $m_{\text{out}}$ increases slightly with mass-loss rate. Most of the mass is contained near the surface of the star where the density sharply peaks but the velocities are small (see Figure (1) panels a) and c)). The ejected mass $m_{\text{ej}}$ is between $10^{-25}$% of the mass-loss rate given that all matter moving with a significant velocity is included in $\dot{M}$, but not all of it is accelerated enough by the radiation force to escape to infinity. $r_{\text{L,edd}}$ is between $\sim 5 - 9 R$ and only $0.001 - 0.02\%$ of the accreted mass is ejected for all models considered. No $m_{\text{ej}}$ point is plotted for the log $\dot{M}$ = 17.25 model as it never reaches the local Eddington luminosity.

In agreement with Figure (3), winds with high mass-loss rate can push more matter out as they have a greater Eddington excess than low mass-loss rate winds. Furthermore, Figure (5) shows that the fraction of $m_{\text{out}}$ that is ejected to infinity increases with mass-loss rate. This variation is more pronounced between the low-end of mass-loss rates ($\sim 10\%$) compared to the high-end of mass loss rates ($<2\%$).

5. SUMMARY AND DISCUSSION

We modeled neutron star winds arising from the large radiation release in type I X-ray bursts. We began by describing the physics that shapes the wind, presenting the general relativistic radiation hydrodynamics equations in a covariant form and outlining the derivation of the final mixed-frame equations that are solved to construct these models.

We then constructed models for a spherically symmetric, steady-state, optically thick wind flowing from a neutron star of $1.4 M_{\odot}$ and a radius of 12 km. Our wind profiles are similar to those of Paczynski & Proszynski (1986) who assumed a heavier composition ($^{56}$Ni) and an inner boundary condition of constant temperature $T = 5$ GK. By matching the wind to a constant column depth at the surface of the star, we find lower base temperatures $\sim 1.4$ GK. Moreover, the photospheric temperatures in our models are $\sim 1$ MK higher than in their models because of our different optical depth requirement at the photosphere. Meanwhile, our models are nearly identical as the ones in Guichandut et al. (2021) up to their photosphere. This is because their flux-limited diffusion treatment reduces to the diffusion approximation at large optical depths. Our models have photospheric radii $>10 R$ and photospheric velocities $<0.006 c$ in accordance with previous GR modeling, and satisfy mass distribution and nuclear energy release requirements for a steady-state solution (Paczynski & Proszynski (1986); Guichandut et al. (2021)).

Our models describe the following process. The high energy released by the X-ray burst elevates the temperature to above a billion kelvin. At these temperatures, the electron scattering cross section is reduced...
and the corresponding opacity decreases significantly; this allows for a large radiation flux to diffuse from the surface of the neutron star. Close to the surface, a thin hydrostatic layer contains most of the mass (Figures (1) and (4)). The radiation force quickly takes over the expansion, and the force exerted by the gas becomes increasingly small in comparison (Figure (2)). The photosphere expands quasi-statically at first, which lowers the temperature, increasing the opacity. Initially, the local luminosity is strongly sub-Eddington, but as the expansion continues with larger velocities the opacity increases, lowering the local Eddington luminosity. Around $\sim 15 - 20$ kilometers after the critical point, the luminosity reaches and exceeds the Eddington luminosity by at most $\sim 0.014 - 0.25\%$, at this point the radiation force ejects matter to infinity (Figure (3)). We find that matter is first ejected at $\sim 5 - 9 R$, and that the ejected mass represents $\sim 10 - 25\%$ of the mass loss rate, $\sim 67 - 83\%$ of the mass present between the critical point and the photosphere, and $\sim 0.001 - 0.02\%$ of the total accreted mass (Figures (4) and (5)).

Our models are limited by the simplifying assumptions initially made. Notably, we assumed that the radiation field is described by the Planck function i.e., $I = S = B$ where $S$ is the source function. This yields an expression for the first and second moments of the radiation field, the flux $F = \sigma T^4$ and radiation energy density $U_{\text{rad}} = aT^4$, respectively, which we used to describe the radiative transfer. Although this approximation is good, it limits our models to the region where the wind is optically thick. As the wind propagates further and becomes optically thin, the gas and radiation are expected to decouple and LTE will no longer apply. Some attempts have been made to model the optically thin regions of the wind: Joss & Melia (1987) assumed a constant luminosity while still allowing the radiation to exert pressure on the gas, while Guichandut et al. (2021) used flux-limited diffusion to describe the radiative transfer. Similarly, steady-state is expected to be a good approximation as the crossing time ($< 1$ s) is significantly smaller than the duration of the burst ($\sim$ few s). Recently, Yu & Weinberg (2018) modeled time-dependent winds and found that their models were qualitatively similar to steady-state models; however, they ignored general relativistic effects which have been shown to be important (Paczynski & Proszynski (1986)). Although all of these previous works provide important insight into the characterization of the winds, we are still in need of a more rigorous treatment of the radiation hydrodynamics with the least assumptions possible. In future work, we intend to drop the assumptions of spherical symmetry, time-independence, and a thermal, isotropic radiation field by solving the radiation hydrodynamics equations (presented in §2.1), within the gray approximation, using the magnetohydrodynamics code Athena ++ (Stone et al. (2020)). This work is, therefore, a basis with which we will test our assumptions by comparing it to models that will explore these dependencies.

6. ACKNOWLEDGEMENTS

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