SPATIAL ARRAY PROCESSING METHODS FOR RADIO ASTRONOMICAL RFI MITIGATION

Brian D. Jeffs and Karl F. Warnick
Brigham Young University
Many array-based instruments suitable for spatial filtering are now in use, and new classes and forms are on the way.
Arrays such as the EVLA, WSRT, GMRT, etc. are well suited for adaptive array processing.

The full array can be configured as a nulling beamformer.

Array elements are the single dishes, or a station beam.

Image credit: Dave Finely, National Radio Astronomy Observatory / Associated Universities, Inc. / National Science Foundation.
Synthesis Array Spatial Filtering

- Post-correlation processing removes interference from visibilities.
- Large aperture means tracking speed is an issue. Need short integration dump times.
- Dish directivity helps and hurts: partially rejects RFI, but reduces INR needed to estimate interference subspace.
- Auxiliary antennas help.
Phased Array Feed Spatial Filtering

- Just now being deployed.
- Small array aperture helps with moving interferer tracking.
- Beamshape distortions may be a critical problem.
- Dish directivity helps and hurts cancelation.
- Must cancel sources outside the FOV in dish sidelobes.
Aperture Arrays

e.g. a LOFAR station aperture array with 96 antennas

Interior view showing four dual pol broadband dipole elements

Images copyright ASTRON
Aperture Arrays

- Low frequency: LOFAR, LWA, MWA, PAPER, etc.
- Parabolic dish collecting area is replaced with a “station” of closely packed simple antennas.
- Wide fields of view for station elements make them highly susceptible to RFI from horizon to horizon.
- Non SOI deep space objects must be treated as RFI and cancelled (or peeled).
- Tracking is slow for station beams, fast for full array.
These have always been promising, but have issues in the RA application
The Narrowband Beamformer

Signal Model:

\[ y(i) = a s(i) + \sum_{d=1}^{D} v_d(i) z_d(i) + n(i) \]

- Repeat for each frequency channel.
- \( w \) is (weakly) frequency dependent.
Calculating $w$ for spatial nulling relies critically on array covariance estimation.

- **Definitions:**
  \[ R = E\{y(i)y^H(i)\} = R_s + R_n + R_z \]

  \[ \hat{R}_k = \frac{1}{N} \sum_{i=kN}^{(k+1)N-1} y(i)y^H(i) \]

- Must identify the interferer vector subspace portion $R_z$

- Must update frequently to track interferer motion
  - Compute $\hat{R}_k$ for all short term integrations (STI), $k$.
  - STI windows are $N_{sti}$ samples long, which depends on motion rate and aperture size.
Classical Adaptive Canceling Beamformers

- **Maximum SNR beamformer**
  - Maximize signal to noise plus interference power ratio:
    \[
    w_{\text{snr}} = \arg \max_w \frac{w^H R_s w}{w^H (\hat{R}_z + \hat{R}_n) w} \quad \rightarrow \quad R_s w_{\text{snr}} = \lambda_{\text{max}} (\hat{R}_z + \hat{R}_n) w_{\text{snr}}
    \]
  - Point source case yields the MVDR solution:
    \[
    w_{\text{mvdr}} = (\hat{R}_z + \hat{R}_n)^{-1} a
    \]

- **LCMV beamformer**
  - Minimize total variance subject to linear constraints:
    \[
    w_{\text{lcmv}} = \arg \min_w w^H \hat{R} w \quad \text{s.t.} \quad C^H w = f \quad \rightarrow \quad w_{\text{lcmv}} = \hat{R}^{-1} C [C^H \hat{R} C]^{-1} f
    \]
  - Direct control of response pattern at points specified by \( C \).
  - Can also constrain derivatives (slope) or eigen vectors.
An Example of MaxSNR canceling

- Very high INR case, +70 dB.
- SNR = +40 dB
- Max SNR output SINR = 50 dB
- 10 element ULA
- Exact covariances
- Output SIR is 139 dB!
Problems with the RA Signal Scenario

- SOI and interferer are well below the noise floor.
  - SNR of -30 dB or worse is common.
  - INR < 0 dB can still severely corrupt SOI.
  - Extremely hard to estimate \( R_z \) from \( R \).

- Motion limits integration time, increases sample estimation error in \( R_z \).

- Weak but troublesome interferers yield shallow nulls.

- Canceling distorts beam beampatterns.
  - Raises confusion limit in sidelobes.
  - Main beam may not have known shape.
Realistic Example of MaxSNR Canceling

- Low INR case, -10 dB.
- SNR = -40 dB
- Input SINR = -40 dB
- Output SIR = -5 dB
- Max SNR output SINR = -30 dB
Pattern Distortion with Moving RFI

- While adapting to null a moving interferer, sidelobe structure is unpredictable.
- Becomes severe as null approaches the main lobe.
- Even sidelobe “rumble” increases confusion noise, hampers on–off subtraction.

- Uniform line array.
- 2 moving interferers, starting at +33 and -35 degrees, then moving to the right.
- 0 dB INR
Improved Methods for RA

Achieving deeper cancelation nulls
The Good News

- Real-time correlators are already needed for all array types (imaging, aperture, PAF)
  - Used to calculate visibilities or calibrate beams.
  - Just need rapid dump times to handle motion
  - Additional computational for spatial filtering is small
- Much progress has been made to address null depth limitations in the RA scenario.
Subspace Projection

- Well suited for synthesis arrays, aperture arrays, PAFs, and post correlation processing.
- Zero forcing, deeper nulls than with total variance minimization.
- Must assume interference is the dominant source.
- Use eigenvector decomposition to identify the interference subspace.
- Partition eigenspace. Largest eigenvalues(s) correspond to interference.

\[
\hat{\mathbf{R}}_k[\mathbf{U}_{\text{int}} | \mathbf{U}_{\text{sig+noise}}] = [\mathbf{U}_{\text{int}} | \mathbf{U}_{\text{sig+noise}}] \Lambda
\]

- Form perpendicular subspace projection matrix:

\[
\mathbf{P}_k = \mathbf{I} - \mathbf{U}_{\text{int}} \mathbf{U}_{\text{int}}^H
\]

- Compute weights and beamform:

\[
\mathbf{w}_{\text{SSP},k} = \mathbf{P}_k \mathbf{w}_{\text{nominal}}, \quad b(i) = \mathbf{w}_{\text{SSP},k}^H \mathbf{y}(i), \quad k = \left[ \begin{array}{c} i \\ N \end{array} \right]
\]
Real-World PAF Subspace Projection

- 19 element L band PAF on Green Bank 20 Meter Telescope.
- Snapshot radio camera image, 21 by 21, 441 simultaneous beams.
- CW interference, hand held antenna and signal generator walking in front of Jansky Lab.

W3OH, no RFI

RFI corrupted image (moving function generator and antenna on the ground)

Adaptive spatial filtering Subspace projection algorithm
Subspace Bias Correction

- Projecting out interference subspace distorts the beampattern.
- Leshem and van der Veen proposed a correction for moving interference over the $N$ sample long term integration (LTI).

$$\tilde{R}_k = P_k \hat{R}_k P_k^H$$

$$\bar{R} = \text{unvec} \left\{ B^{-1} \text{vec} \left( \sum_{k=0}^{J-1} \tilde{R}_k \right) \right\}$$

$$B = \sum_{k=0}^{J-1} P_k^* \otimes P_k, \quad J = \left[ N / N_{\text{sti}} \right]$$

- Well suited for satellite RFI at large imaging arrays.
- Use $\bar{R}_j$ directly as an imaging visibility matrix.
- RFI is removed from each STI, but in $\bar{R}_j$ no subspace is missing.
Auxiliary Antenna Methods

- Improve interference subspace estimate $\hat{R}_z$ and increase null depth.
- Aux antennas return higher INR signal.
- Must track moving interferers.
- One antenna per interferer.
Cross Subspace Projection

- Extend array vector to include auxiliaries.

\[ y(i) = \begin{bmatrix} y_m(i) \\ y_a(i) \end{bmatrix} \]

\[ R = \begin{bmatrix} R_{mm} & R_{ma} \\ R_{am} & R_{aa} \end{bmatrix} \]

- Compute “projection” matrix with SVD on \( \hat{R}_{ma} \)

\[ \hat{R}_{ma} = U \Sigma V^H \]

\[ U_s = [u_{D+1}, \ldots, u_{M_m}] \]

\[ P_{CSP} = [U_s U_s^H, 0_{M_m}] \]

\[ \hat{R}_{CSP} = P_{CSP} \hat{R} P_{CSP}^H \]

VLA simulation for two stationary interferers and two small auxiliary dish antennas. Source is 1 Jy OH emission. INR at primary feeds is 146 dB above the plotted dBm interferer power level.
Bias Corrected Array PSD Estimation

- When averaging is performed for a beamformed PSD estimator, or total power observation, beampattern distortion can be completely corrected.

- Based on subspace projection beamforming.

\[ P_k = I - U_{\text{int}} U_{\text{int}}^H, \quad \hat{R}_k [U_{\text{int}} \mid U_{\text{sig+noise}}] = [U_{\text{int}} \mid U_{\text{sig+noise}}] \Lambda \]

\[ B = \frac{1}{J} \sum_{k=0}^{J-1} P_k^* \otimes P_k, \quad J = \left[ \frac{N}{N_{\text{sti}}} \right] \]

\[ \hat{S}_{y,c} = \frac{\alpha}{J} (w^H \otimes w^T) B^{-1} \sum_{k=0}^{J-1} (P_k \otimes P_k^*) \left( (Y_k GF) \circ (Y_k GF)^* \right) \]

Where \( F \) is the Fourier transform matrix, \( G \) is a diagonal window weighting matrix (e.g. Hamming window), \( \alpha = 1/(\text{diag}\{G\}^T \text{diag}\{G\}) \), and

\[ Y_k = \left[ y(k N_{\text{sti}}), y(k N_{\text{sti}} + 1), \cdots, y([k+1] N_{\text{sti}} - 1) \right] \]
Bias Corrected Array PSD Results

Power Spectral Density using all data via Welch’s method

- Subspace Proj
- Conventional
- Bias corrected subspace

2 interferers, weak SOI, $J = 900$, 512-point STIs
Bias Corrected Array PSD Results (cont.)

- Time varying adaptive beampattern.
- Effective beampatterns as seen by the PSD estimators (i.e. these are averaged over all STIs due to the PSD calculation.)
- Bias corrected PSD matches the conventional fixed beam response, even while canceling interference.
Conventional SSP Limitations

- Detailed simulation
- 19-element PAF on 20m reflector, 0.43f/D
- Correlated spillover noise, mutual coupling, modeled 33K Ciao Wireless LNAs.
- Measured array element radiation patterns.
- Physical Optics, full 2D integration over reflector.

Subspace estimation error due to sample noise from short STI
Subspace smearing error due to motion, with no sample estimation error.
**Low Order Parametric Models for SSP**

- Fit a series of STI covariances $\hat{R}_k$ to a polynomial that can be evaluated at arbitrary timescale.
  - Beamformer weights can be updated *every* time sample.
  - Use entire data window to fit polynomial for less sample estimation error.

- Minimize the squared error between STI sample covariances and the polynomial model $C_{LS}$:

$$C_{LS} = \arg\min_{C} \sum_{k=1}^{K} \left\| \hat{R}_k - \tilde{R}_{int}(t_k, C) \right\|_F^2,$$

where $t_k = kN_{sti}T_s$

$$\tilde{R}_{int}(t, C) = a_{poly}(t, C) a_H^{poly}(t, C)|_{t=nT_s},$$

where $C = [c_0 \cdots c_r]$

$$a_{poly}(t, C) = c_0 + c_1 t + \cdots + c_r t^r$$
Real Data Cancelation Results

Tx near boresight, Slow Motion
INR $-12\,\text{dB}$, Poly = 8, with Pre-Whitening

Tx in sidelobes, Slow Motion
INR $-12\,\text{dB}$, Poly = 8, with Pre-Whitening
Real Data Cancelation Results

Tx near boresight, Slow Motion
INR $-12\,\text{dB}$, Poly = 8, with Pre–Whitening

Tx in sidelobes, Slow Motion
INR $-12\,\text{dB}$, Poly = 8, with Pre–Whitening
Final Observations
Additional Methods

- Robust beamforming
  - Helpful if array calibration has errors. Keeps SOI from being canceled.

- Spectral scooping correction
  - Block mode covariance matrix processing for beamformer weight updates.
  - Narrowband RFI.
  - Surprising behavior: the time-varying spatial filter places a spectral null in PSD estimates.
  - The cause of this bias has been described analytically.
  - A straightforward correction has been proposed.
Progress Towards Adoption

- Parks Telescope adaptive filter for digital TV may be the only current production system.
- Despite significant promise and potential, adoption has been slow.
- Perhaps astronomers are reluctant to move from the tried and true.
- We have not yet identified the critical science case.
- Computational and infrastructure costs are incremental, but non-trivial.
- I hope presented progress in overcoming spatial filtering limitations will spur adoption.
Conclusions

- RFI spatial Filtering for radio astronomy offers a new mode of mitigation beyond time blanking, frequency excision, and avoidance.
- It is challenging due to low INR and SNR.
- Algorithms common to wireless comm., radar, and sonar do not drive deep enough nulls, and distort beampatterns.
- Several algorithm enhancements have been proposed which may solve these limitations.
- Widespread adoption of these promising methods may take some time for confidence to build.
- The “critical science case” which cannot be pursued without spatial filtering would spur deployment.


